Труды МАИ. 2023. № 130
Trudy MAI, 2023, no. 130

Научная статья
УДК 531.3
DOI: 10.34759/trd-2023-130-21

ОПТИМИЗАЦИЯ И МОДЕЛИРОВАНИЕ ПОСАДОЧНОЙ ТРАЕКТОРИИ ДЛЯ МИНИ-БПЛА С УЧЕТОМ ОГРАНИЧЕНИЙ НА УПРАВЛЕНИЕ И ПОСАДОЧНУЮ СКОРОСТЬ

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Аннотация. В статье представлен метод оптимизации посадочной траектории мини-БПЛА с учетом ограничений на управление и посадочную скорость, а также способ сопровождения найденной траектории. Выбранными оптимальными управлениями являются именно нормальная и тангенциальная перегрузки. Критерий оптимальности представлен в виде Больца, включающая точность приземления и расхода топлива

БПЛА. Применение принципа максимума Понтрягина превращает задачу оптимального управления в краевую задачу, которая решается методом продолжения по параметру. Чтобы проверить оптимальную траекторию, конкретный тип БПЛА выбран для моделирования сопровождения вышеупомянутой траектории с помощью программного обеспечения Matlab Simulink. Результаты показывают, что применение алгоритма сопровождения оптимальной траектории обеспечивает точность и безопасность посадки БПЛА.

Ключевые слова: оптимизация траектории, метод продолжения по параметру, нормальная перегрузка, тангенциальная перегрузка, алгоритма сопровождения траектории, ограничение на управление

Для цитирования: Нгуен Н.Д., Доан Т.Т., Нгуен Т.Т., Чан К.М., Нгуен К.А., Нго В.Т. Оптимизация и моделирование посадочной траектории для мини-БПЛА с учетом ограничений на управление и посадочную скорость // Труды МАИ. 2023. № 130. DOI: 10.34759/trd-2023-130-21

Original article

LANDING TRAJECTORY OPTIMIZATION AND SIMULATION FOR MINI-UAV CONSIDERING CONSTRAINT OF CONTROL AND LANDING SPEED

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Abstract. This article presents the method of optimizing the landing trajectory of mini-UAV considering constraint of control and landing speed and the method of tracking the found trajectory. The chosen controls are namely normal and tangential overload. The selected objective function is in the Bolza form, including landing accuracy and energy consumption during flight. Applying the Pontryagin's maximum principle turns the optimal control problem into the boundary problem, which is solved by the parameter continuation method. To verify the optimal trajectory being gained, the authors selects a specific type of UAV to simulate tracking on the aforementioned trajectory through the Matlab Simulink software. The results show the application of the optimal trajectory tracking controller assures the accuracy and safety of UAV landing.

Keywords*:* trajectory optimization, parameter continuation method, normal overload, tangential overload, trajectory tracking controller, Constraint of control

For citation: Nguyen Ngoc Dien, Doan The Tuan, Nguyen Thanh Tung, Tran Quang Minh, Nguyen Quang Anh, Ngo Van Toan. Landing trajectory optimization and simulation for

mini-UAV considering constraint of control and landing speed. *Trudy MAI*, 2023, no. 130. DOI: 10.34759/trd-2023-130-21

1. PROBLEM STATEMENT

The recent year have witnessed numerous studies on designing UAV landing trajectory [1,2,5,6,7,8] since landing process entails tremendous circumstances; thus, drones are obligated to comply with several referential trajectories to attain efficient landing with respect to some performance indices [1,8]. It is possible to employ analytical and numeral method in designing landing trajectory, in which analytical method holds an advantage of attaining analytical control. Nonetheless, finding the solution is of hinderance and often has to simplify the problem. As for the numerical method, its merit is to finding the trajectory quickly, but in real-time, challenges awaits especially in the case of factors affecting the UAV such as wind disturbance.

Controls are regularly chosen with angle of attack, pitch angle or normal overload... these controls are restricted and bounded within a range of definite values; however, there go few studies considering control constraint for mini UAVs in the process of finding optimal control [2, 6, 7]. Regarding mini-sized UAVs, the control (normal overload) falls within in a narrow range of values would engender difficulty to the optimal control by analytical method. Therefore, the authors utilize numerical method to find the optimal control. The application of Pontryagin's maximum principle allows to transform the optimal control problem to the boundary one, which takes into account the limitation of the control. The boundary conditions of the problem takes into account the position, attitude and speed of UAV when landing. The researchers handle the boundary problem by the parameter continuation method, which outweighs other methods by allowing the method's parameters to be included in the object equation, it helps to increase increase the convergence ability of the method in case of passing the special point if necessary [3, 4, 15]. After finding the optimal trajectory, it is in need of establishing an optimal trajectory tracking controller. The essence of the tracking control system is to eliminate the deviation between the instant and the programed trajectory. Therefore, This paper aims to solve the following main contents:

First, derivation of dynamic model of UAV vertical motion;

Second, developing a landing trajectory for the UAV;

Third, synthesizing the landing trajectory tracking algorithm for UAVs;

Fourth, simulation and evaluation of results.

The results of this paper are the optimal trajectories in some circumstances and the verification of feasibility of following these trajectories, thereby offering recommendations to the UAV user about the appropriate selection of landing of trajectory.

2. PROBLEM SOLUTION

2.1. Derivation of dynamic model of UAV vertical motion

The UAV vertical motion is defined to occur only in the vertical plane, means that the trajectory plane Ox_ky_k coincides with Ox_oy_o plane shown in Figure 1.

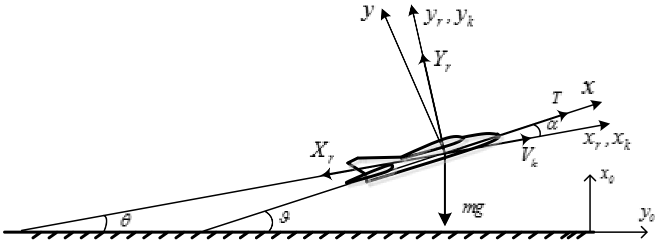

Figure 1 – UAV motion in vertical plane

In the absence of wind disturbances, the irregular vector $\bar{V}_r$ coincides with the velocity vector $\bar{V}_k$ ($\bar{V}_r \equiv \bar{V}_k$), and the equations of UAV vertical motion is presented as follow:

$$m\left(\frac{dV_k}{dt}\right) = T\cos\alpha - C_x(\alpha).\frac{\rho.V_r^2}{2}.S - G\sin\theta \tag{1}$$

$$mV_k\frac{d\theta}{dt} = T\sin\alpha + \left(C_y(\alpha) + C_y^{\varpi_z}.\omega_z.\frac{b_a}{V_r} + C_y^{\delta_c}.\delta_c + C_{ydng}\right).\frac{\rho.V_r^2}{2}.S - G\cos\theta \tag{2}$$

$$J_z\left(\frac{d\omega_z}{dt}\right) = (m_z^{\delta_c}.\delta_c + m_z^{\varpi_z}.\omega_z.\frac{b_a}{V_r} + m_{zo} + m_{z\alpha} + m_{z\_dng}).\frac{\rho.V_r^2}{2}.S.b_a - T.h_{dc} \tag{3}$$

$$\frac{dx_o}{dt} = V_k\cos\theta \tag{4}$$

$$\frac{dy_o}{dt} = V_k\sin\theta \tag{5}$$

$$\frac{d\vartheta}{dt} = \omega_z \tag{6}$$

$$\theta = \vartheta - \alpha \tag{7}$$

In which: m – mass of UAV; T – engine thrust; $C_x(\alpha)$ - drag coefficient with respect to angle of attack; $C_y(\alpha)$ - Lift coefficient depends on angle of attack; $C_y^{\varpi_z}$ - derivative of lift coefficient depends on angular velocity ϖ_z with $\varpi_z = \omega_z\frac{b_a}{V_r}$; $C_y^{\delta_c}$ - derivative of lift coefficient depends on elevator deflection; C_{ydng} - lift coefficient of horizontal tail; b_a – mean aerodynamic chord; S - UAV's wing area; G – gravity; $\frac{\rho.V_r^2}{2}$ - dynamic pressure; $m_{z\alpha}$ - moment coefficient depens on angle of attack; $m_z^{\varpi_z}$ - the derivative of the pitching moment coefficient depens on ϖ_z; $m_z^{\delta_c}$ - the derivative of the pitching moment coefficient depens on elevator deflection; m_{z_dng} - moment coefficient depens on the deviation of the horizontal tail; m_{zo} - initial moment coefficient depens on the unsymmetrical shape of the UAV relative to the horizontal plane; h_{dc} - engine‘s height relative to the UAV longitudinal axis.

2.2. Developing landing trajectory of UAV

Figure 2 depicts the diagram of closed-loop UAV landing controller as follow:

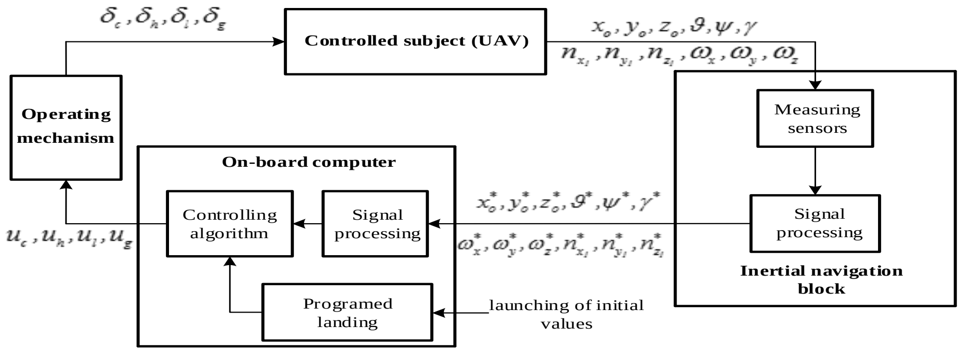

Figure 2 – Block diagram of closed-loop UAV landing controller

In which:

x_o, y_o, z_o - UAV position in ground coordinate system $O_ox_oy_oz_o$;

$n_{x_1}, n_{y_1}, n_{z_1}$ - overload relative to axes of the integrated coordinate system $Ox_1y_1z_1$;

$\omega_x, \omega_y, \omega_z$ - rotational velocity relative to axes of the body frame $Ox_1y_1z_1$;

x_o^*, y_o^*, z_o^* - UAV position measured by inertial navigation block;

$n_{x_1}^*, n_{y_1}^*, n_{z_1}^*$ - overload measured by inertial navigation block;

$\omega_x^*, \omega_y^*, \omega_z^*$- rotational velocity relative to axes of the body frame measured by inertial navigation block;

u_c, u_h, u_l, u_g- signals controlling elevator and rudder deflection, aileron deflection and throttle

$\delta_c, \delta_h, \delta_l, \delta_g$ - deflection of, rudder, aileron and throttle control level.

When considering UAV like point-mass, the equations of UAV vertical motion (1) $\div$ (7) are given by:

$$\dot{V} = g(n_x - \sin\theta); \dot{\theta} = \frac{g}{V}(n_y - \cos\theta); \dot{x} = V\cos\theta; \dot{y} = V\sin\theta, \qquad (8)$$

In which: V - UAV velocity; θ - flight path angle; x - horizontal range; y - altitude; g – gravitational acceleration (g = 9,80665 m/s²); $X = [V, \theta, x, y]^T$ - state vector of UAV; n_x, n_y - tangential overload and normal overload.

Choosing control vector $u = [n_x, n_y]^T$., the objective function under Bolza form, is given as:

$$J = 0,5\rho_1(\mathrm{x}(\mathrm{t}_f) - x_f)^2 + 0,5\rho_2(y(\mathrm{t}_f) - y_f)^2 + 0,5\rho_3(\theta(\mathrm{t}_f) - \theta_f)^2$$
$$+ 0,5\rho_4(V(\mathrm{t}_f) - V_f)^2 + 0,5\int_{t_0}^{t_f} u^T k^{-2} u dt \qquad (9)$$

In which: $\rho_1, \rho_2, \rho_3, \rho_4$ - constants; $k^2 = diag(k_1^2, k_2^2)$ - coefficient; t_0 and t_f initial time and terminal time of control process; V_f, θ_f, x_f, y_f - desired value at terminal time t_f; $V(t_f), \theta(t_f), x(t_f), y(t_f)$ - output value at terminal time t_f

Then, the equivalent Hamilton function is developed as:

$$H = P_V g(\mathrm{n}_x - \sin\theta) + P_\theta \frac{g}{V}(n_y - \cos\theta) + \mathrm{P}_x V\cos\theta + P_y V\sin\theta + \frac{1}{2}u^T k^{-2} u. \quad (10)$$

The optimal control is gained in each point causing the Hamiltonian function H to reach its maximum.

In this paper, it is assumed that the tangential overload n_x is unrestricted, with the optimal condition $\frac{\partial H}{\partial n_x}=0$, the optimal control is: $n_x=-P_V g k_1^2$. By Pontryagin ‘s maximum principle $\max_{n_y \in N_y} H(x^*, n_x^*, n_y, P^*, t) = H(x^*, n_x^*, n_y^*, P^*, t)$, the optimal control n_y is gained; hence, the UAV ‘s motion equations are given by:

$$\dot{V} = g(n_x - \sin\theta); \tag{11}$$

$$\dot{\theta} = \frac{g}{V}(n_y - \cos\theta); \tag{12}$$

$$\dot{x} = V\cos\theta; \tag{13}$$

$$\dot{y} = V\sin\theta; \tag{14}$$

$$\dot{P}_V = -\frac{\partial H}{\partial V} = P_\theta \frac{g}{V^2}(n_y - \cos\theta) - \mathrm{P}_x \cos\theta - P_y \sin\theta; \tag{15}$$

$$\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = P_V g\cos\theta - P_\theta \frac{g}{V}\sin\theta + \mathrm{P}_x V\sin\theta - P_y V\cos\theta; \tag{16}$$

$$\dot{\mathrm{P}}_x = -\frac{\partial H}{\partial x} = 0; \tag{17}$$

$$\dot{\mathrm{P}}_y = -\frac{\partial H}{\partial y} = 0; \tag{18}$$

It is of significance to find the initial conditions $P_V(t_0)$, $P_\theta(t_0)$, $P_x(t_0)$, $P_y(t_0)$, to match the boundary ones $V(t_f)=V_f$, $\theta(\mathrm{t}_f)=\theta_f$, $\mathrm{x}(\mathrm{t}_f)=x_f$, $y(\mathrm{t}_f)=y_f$, $H(X,P,t_f)=0$. Indeed, this aims to solve the boundary problem, which may encounter numerous challenges owing to the relation to computation time, the choice of initial approximate parameters and the convergence of the method. Some studies suggest using the Newton-Raphson method

[13, 20]. Nevertheless, given the constraints of control, it is arduous for the application of Newton-Raphson method [15, 16]. Other research posits the continuation method of solving by parameters [3, 4, 17, 18], this method has demonstrated its preeminence.

The authors consolidate the theory via a specific case which considers initial state of UAV with following values $V(0) = 50\ m/s$; $\theta(0) = 0\ radian$; $x(0) = 0\ m$; $y(0) = 60\ m$ and desired terminal state of UAV: $V_f = 30\ m/s$; $\theta_f = 0\ radian$; $x_f = 500\ m$; $y_f = 0\ m$.

Mini UAV type chosen for simulation and evaluation is the UAV-70V. This is a small-sized UAV researched and manufactured by the Viet Nam Aerospace Association is often used for remote monitoring. To ensure the UAV-70V's tail does not hit ground when landing, the pitch angle must be $\vartheta \le 12^o$; thus, the author has considered the constraint of the normal overload n_{yhc} [10]. Which limits angle of attack (α) and pitch angle angle (ϑ). In the optimizing trajectory algorithm, the value of pitch angle at final time is computed not to exceed limit value 10^o for the purpose of error margin while controlling UAV.

After solving the above equations, the program trajectory of the UAV is attained as H_{ct}, $\theta_{ct}, \vartheta_{ct}, V_{ct}$ shown in Figure 3, Figure 4, Figure 5, Figure 6, as well as tangential overload, normal overload of UAV shown in Figure 7, Figure 8 presented in blue line. When changing the initial altitude on landing, the results also match the program trajectory as well as the UAV's normal and tangential overload as the red and yellow lines.

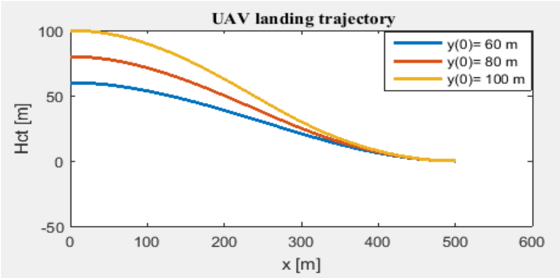

Figure 3 – Landing trajectory of UAV

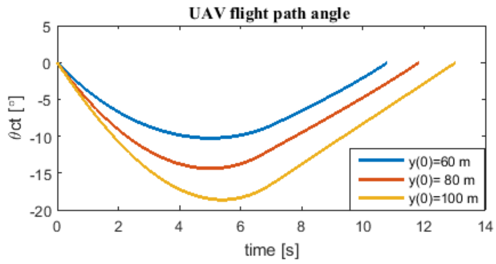

Figure 4 – Flight path angle of UAV

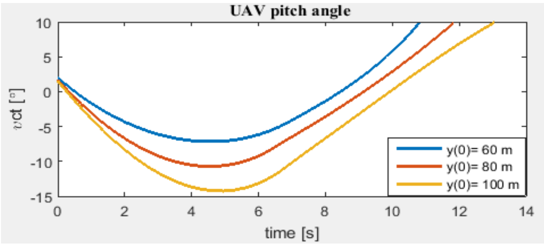

Figure 5 – Pitch angle of UAV

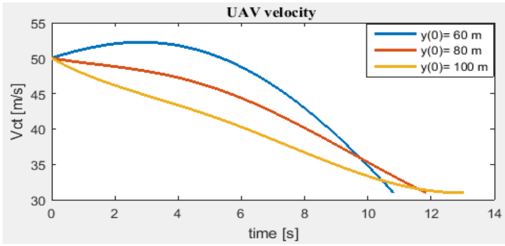

Figure 6 – UAV 's velocity

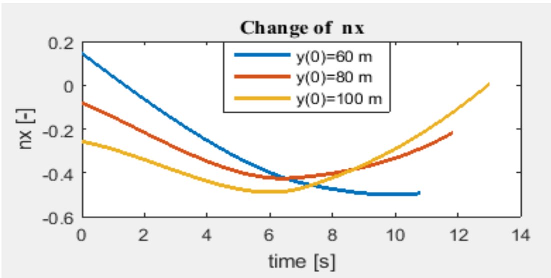

Figure 7 – Tangential overload

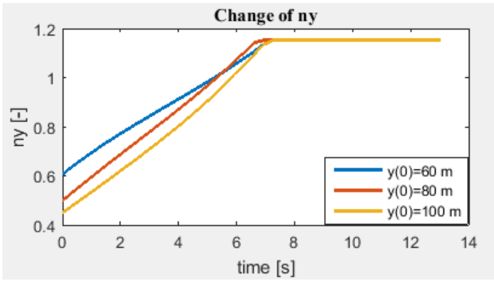

Figure 8 – Normal overload

2.3. Tracking algorithm of the UAV landing trajectory

Via the equations of vertical motion of the UAV (equations (1) ÷ (7)), in order to control UAV under predetermined programs of trajectory and speed, it is required to develope a closed-loop of altitude and speed.

The relationship between UAV's altitude and flight path angle burgoens the derivative of altitude as

$$\dot{H} = V_k . \sin\theta \tag{19}$$

On the other hand, with $n_y = \frac{N}{G} = \frac{P + Y}{G}$, it is inferred that the change of N would lead to the change of normal overload n_y and consequently change the flight altitude.

N = P + Y – control force;

G = mg – UAV mass;

Remarks: Therefore, controlling UAV tracking trajectory will be accomplished by controlling flight altitude or pitch angle of UAV (flight path angle of UAV) and controlling normal overload n_y *of UAV.*

2.2.2. Tracking trajectory using pitch angle control

The pitch angle tracking algorithm is found by:

$$\delta_c = K_{\delta_c}^{\omega_z}.\omega_z + K_{\delta_c}^{\vartheta}.\left(\vartheta - \vartheta_{ct}\right) \tag{20}$$

To track given flight altitude H_{ct}, it is required to make pitch angle control with respect to altitude ϑ_{ct}

$$\vartheta_{ct} = K_{\vartheta}^{H}(H_{ct} - H) \tag{21}$$

Substituting (21) in (20), the altitude control depends on the pitch angle:

$$\delta_c = K_{\delta_c}^{\omega_z}.\omega_z + K_{\delta_c}^{\vartheta}.\vartheta + K_{\delta_c}^{H}.\left(H - H_{ct}\right) \tag{22}$$

In which: $K_{\delta_c}^{\omega_z}.\omega_z$ - damping control; $K_{\delta_c}^{\vartheta}\vartheta$ - control depends on the pitch angle; $K_{\delta_c}^{H}\left(H - H_{ct}\right)$ - main control depends on prefixed altitude deviation (H_{ct}) compared to instantaneous altitude (H) during flight for in-trajectory maintenance.

In addition, trajectory tracking algorithm controlling pitch angle can be used:

- Method of using integral term in control algorithm

$$\delta_c = K_{\delta_c}^{\omega_z}\omega_z + K_p\left(\vartheta - \vartheta_{ct}\right) + \frac{K_i}{P}\left(\vartheta - \vartheta_{ct}\right) \tag{23}$$

When employing control algorithm (23), closed loop system eliminates the residual steady-state error that occurs from moment disturbance, wind disturbance and measurement error of sensors ω_z, ϑ.

- Method of using equal feedback in control algorithm

$$\delta_c = K_{\delta_c}^{\omega_z}\omega_z + \frac{T_q P}{T_q P + 1}\left(K_p(\vartheta - \vartheta_{ct}) + \frac{K_i}{P}(\vartheta - \vartheta_{ct})\right) \tag{24}$$

Obviously, the main cause of altitude error in steady mode is pitch error $\Delta\vartheta$. Therefore, the pitch error impacting on system must be eliminated, which is implemented by utilizing the filter of equal feedback in the closed loop ϑ. It also means to ensure $\vartheta_{xl} = 0$ to maintain $\Delta H_{XL} = 0$.

2.2.3. Using normal overload in tracking algorithm

In flight, the UAV is subjected to 4 forces: engine thrust *($\bar{T}$), lift force* ($\vec{Y}$); gravity force $\bar{G} = m\bar{g}$ and control force. To control altitude during flight, it requires change magnitude and direction of these forces. During flight, $\bar{G}$ is almost unchanged while $\bar{T}$ and $\bar{Y}$ constant change. Sum of two vector $\bar{N} = \bar{Y} + \bar{T}$, which is so-called control force. Normal overload can be defined as:

$$n_y = \frac{\bar{N}}{\bar{G}} \tag{25}$$

Control algorithm employed normal overload can be expressed as:

$$\delta_c = K_{\delta_c}^{\omega_z}\omega_z + K_{\Delta n}\left(n_y - n_{yct}\right) \tag{26}$$

in which: $n_{yct} = K_{n_y}^{\vartheta}\left(\vartheta - \vartheta_{yct}\right)$

So, it is inferred that expression of control

$$\delta_c = K_{\omega_z}.\omega_z + K_{n_y}.n_y + K_p\left(\vartheta - \vartheta_{ct}\right) \tag{27}$$

2.2.4. Combined trajectory tracking control

Controlling the UAV tracking program trajectory can be achieved with above control algorithms. In practie, we can use algorithm which combines all control to improve accuracy. Thus, the combined control can be expressed as:

$$\delta_c = K_{\omega_z}\omega_z + K_p(\vartheta - \vartheta_{ct}) + \frac{K_i}{P}(\vartheta - \vartheta_{ct}) + K_{ny}(n_y - n_{yct}) + K_H(H - H_{ct}) \tag{28}$$

In a nutshell, it is apparent that the aforementioned control algorithms have their own merits and drawbacks. Through the study process, the control algorithm (28) employing PI term and normal overload control embodies distinct prominence. Within the scope of this paper, the authors conduct simulation control algorithm (28) to evaluate performance of tracking controller and simultaneously evaluate the ability to implement the landing trajectory optimization algorithm in a specific UAV.

3. SIMULATION RESULTS

The type of mini UAV simulated and surveyed is the "UAV-70V". The geometrical features and mass - inertia - centering characteristics of the UAV were identified directly from the three-dimensional drawing of the UAV by INVENTOR software. Aerodynamic characteristics: The aerodynamic coefficients are determined by ANSYS CFX software, based on solving the system of Navier-Stock equations by the finite volume method [11]. The efficiency coefficients of rudder ($m_z^{\delta_c}, m_x^{\delta_l}, m_y^{\delta_h}$) and other aerodynamic derivatives ($m_z^{\varpi_z}$, $m_x^{\varpi_x}, m_y^{\varpi_x}, m_y^{\varpi_y}, \dots$) are calculated by linear discrete vortex method [12].

It is assumed that the UAV is equipped with ideal sensors measuring its position, velocity, altitude in space, angular velocities and overloads without static and dynamic errors, i.e. all sensors that measure the motion parameters of the UAV are simulated by ideal amplifiers. In addition, it is assumed that the UAV is equipped with actuators in the form of stepless electric actuators which has maximum angular rate of control surface, not exceeding 200 degrees/s, and their inertia is simulated by an inertial phase with a time constant $T_{qt} = 0,03 s$

It is assumed that initial position of the UAV is at point A before landing (Figure 8) with its velocity $V(0) = 50\ m/s$, initial flight path angle $\theta(0) = 0\ radian$, then the location of the UAV on initial landing is $y(0) = 60\ m$; $x(0) = 0\ m$.

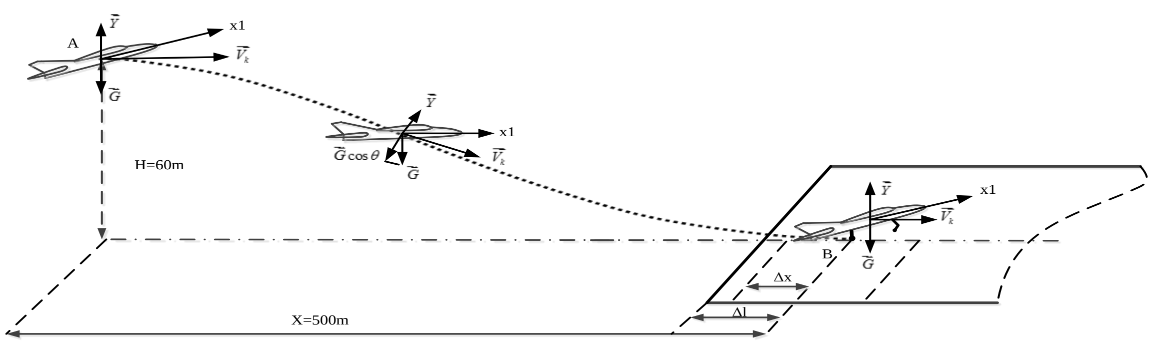

Figure 9 – UAV's state when landing

In which: L- runway length; ΔL - distance from the beginning of the runway to the desired landing position ($\Delta L = 40\ m$).

It is required to ensure the following conditions when the UAV hits the ground:

- Altitude error at the time of landing: $0 m \le |\Delta y| \le 0,3 m$;

- Distance error: $|\Delta x| \leq 30\,m$;

- Landing speed $V_{hc} \geq V_{\min}$.

In which: $V_{\min}$ is achieved from the condition of balancing between the UAV ‘s weight and lifting force when landing (at the moment just before landing caused by the normal reaction force of ground acting on UAV).

$$Y = C_{yHC}\frac{\rho V_{\min}^2}{2}S = G \qquad (29)$$

So: $$V_{\min} = \sqrt{\frac{2G}{C_{yHC}\rho S}} \qquad (30)$$

In which: C_{yHC} - lift coefficient at the moment of landing;

ρ - air density at ground.

- Vertical velocity when landing: $|V_y| \leq 1\,m/s$;

- UAV’s pitch angle when landing: $0 \leq \vartheta \leq 12^o$;

- UAV’s normal overload: $-1 \leq n_y \leq 3,5$. Particularly, UAV’s normal overload when landing must be greater than 1.

Through the dynamic model of the UAV (equations (1) ÷ (7)), the characteristic parameters of the UAV and control law (28), the authors establish a simulation model of a closed loop control of UAV vertical motion using Matlab Simulink software shown in Figure 10. In which, the values of teta_mm1, Ny_mm1, H_mm1 in the Pitch angle controller are program pitch angle, program normal overload, and program altitude are achive from optimal landing trajectory solution.

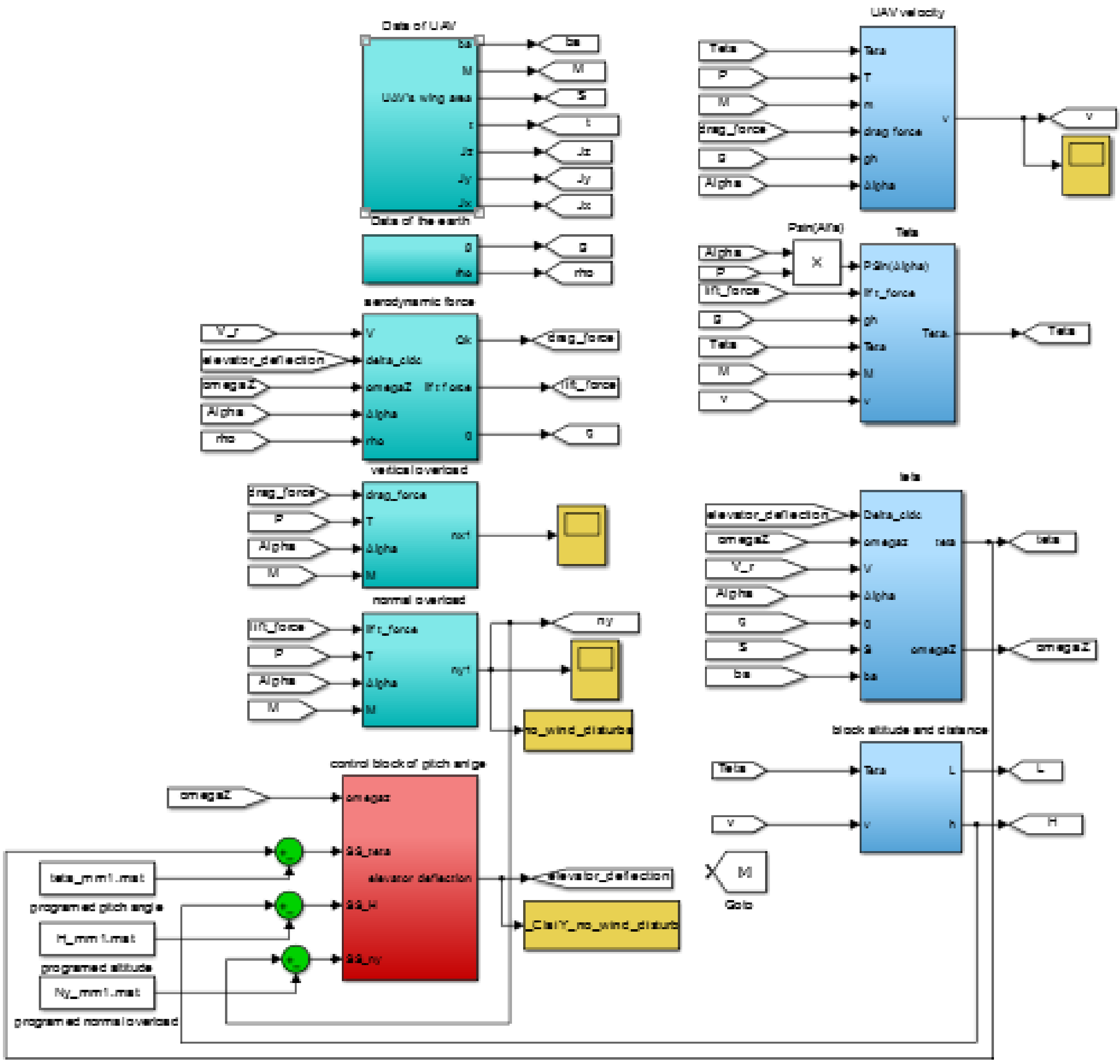

Figure 10 – Block diagram of closed loop of the UAV vertical motion

In pitch angle controller shown in Figure 11, using Simulink Response Optimization tool in Simulink, the results would contain the coefficients K_p, K_i - corresponding to the scaling and integration coefficients of PI controller as well as $K_{\omega z}$, K_{ny} - corresponding to damping coefficients and overload error exclusion coefficients.

Foremostly, the coefficients K_p , K_i, $K_{\omega z}$, K_{ny} must be appropriately selected to conduct control law (28). The law's aim is to reach $\Delta\partial \to 0$, $\Delta H \to 0$ and $\Delta n_y \to 0$, however, the attainment must satisfy the limitation on the angle of attack and the overload factor (

$|\alpha| \leq 12^o$ và $-1 \leq n_y \leq 3,5$), which means UAV must fly within acceptable angle of attack and overload limit.

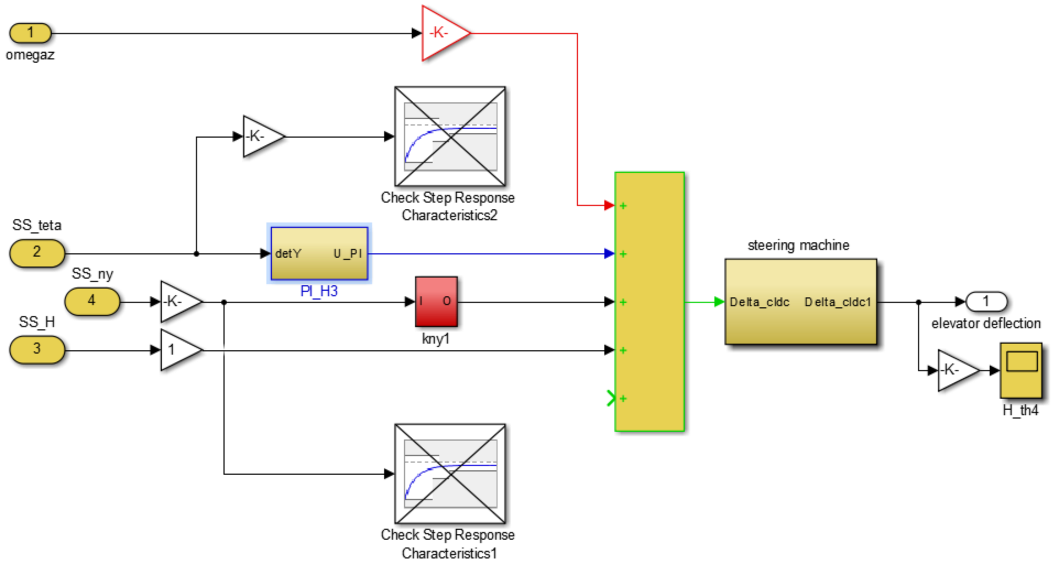

Figure 11 – The UAV 's pitch angle controller

In light of employing Matlab – Simulink's optimizing tool namely "Simulink Design Optimization", the researchers can adjust the desired response signals in Simulink model by adding the "Check step Response Characteristics" block in simulation model. The findings of coefficient values are presented in Figure 12.

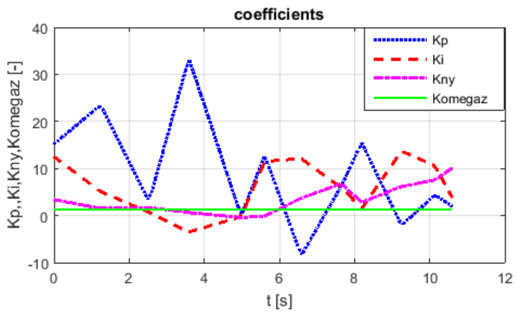

Figure 12 – Coefficients K_p , K_i , $K_{\omega z}$, K_{ny}

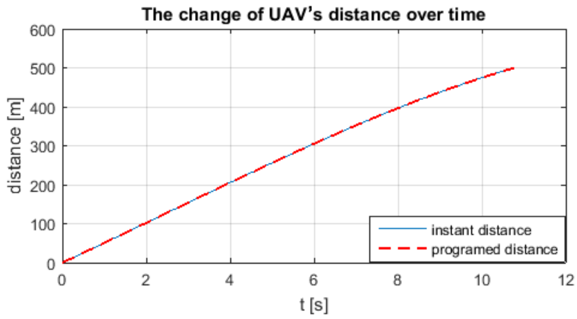

Figure 13 – The change of UAV's distance over time

Running simulation program with obtained coefficients in Figure 12 indicates that the system controls the landing in program trajectory with high accuracy. Altitude error at the final time (when landing) (Figure 14) $\Delta H = 0{,}12\, m$, distance error $\Delta x = 0{,}3\, m$ (Figure 13) which meet the requirement for UAV's safety landing. UAV's pitch and flight path angle contiguously follow the programed pitch and flight path angle (Figure 16). The flight path angle $-0{,}3^o$ and pitch angle $10{,}1^o$ when the UAV lands are guaranteed to be within the acceptable range. The normal overload of the UAV (Figure 18) follows the programed normal overload and is within the allowable range.

Figure 17 displays the change of elevator deflection angle is appropriate within the limit range $\pm 25^o$. Whereas the figure 19 depicts the change of UAV's attack angle which falls within the acceptable limit range and it reached $10{,}4^o$ at the final time.

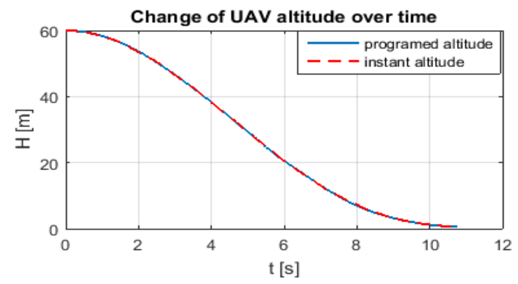

Figure 14 – Change of UAV altitude over time

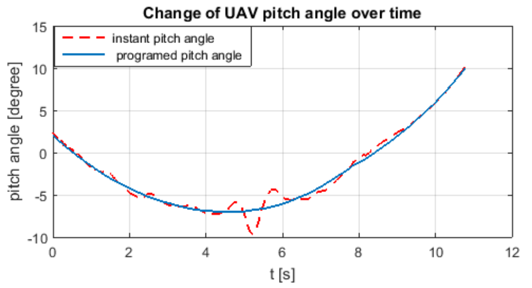

Figure 15 – Change of UAV pitch angle over time

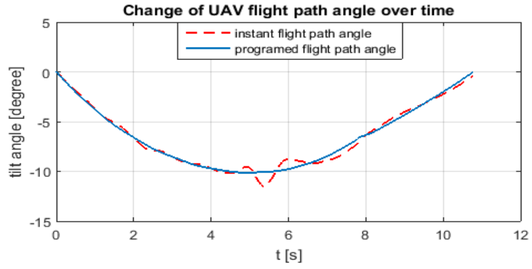

Figure 16 – Change of UAV flight path angle over time

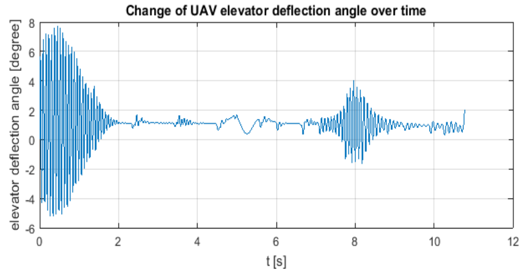

Figure 17 – Change of UAV elevator deflection angle over time

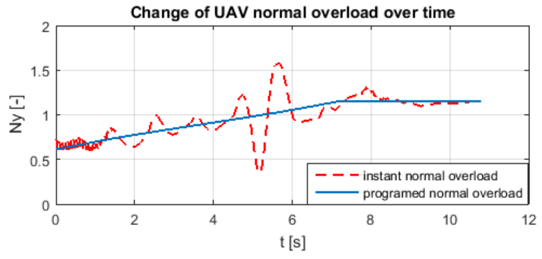

Figure 18 – Change of UAV normal overload over time

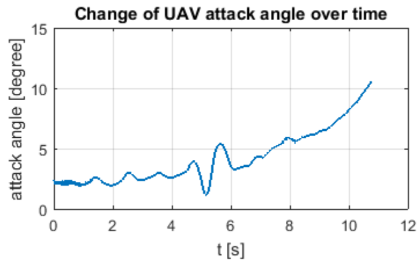

Figure 19 – Change of UAV attack angle over time

4. CONCLUSION

This article copes with the optimal landing trajectory of mini UAV considering control restriction. The significane of this study is that the landing speed is smaller than the initial one. The maximum principle Pontryagin allows the reseachers to turn the optimal control problem into the boundary problem, which is solved by the parameter continuation method. The optimal trajectory is evaluated and verified by the Matlab Simulink software. The result affirms that the dynamic model and the control algorithm tracking the optimal trajectory as built and selected are of appropriateness. Using the proposed control algorithm, it is possible to control the UAV to track the optimal trajectory, ensuring the safe landing. Hence, it is possible to apply the research findings in the practical design and manufacture of the landing control system as well as the landing control process of the UAV. Nonetheless, the paper using numerical method to find the optimal trajectory argues that the achieved trajectory may become a recommendation in case of pre-planned landing or necessity of meeting the real-time requirements or with disturbance, the analytical method would outweigh the numerical one in use.

Список источников

1. Horla D., Cieślak J. On obtaining energy-optimal trajectories for landing of UAVs // Energies, 2020, vol. 18 (8), pp. 2062. DOI:10.3390/en13082062

2. Rucco A., Aguiar A.P., Hauser J. Trajectory optimization for constrained UAVs: A virtual target vehicle approach // In 2015 International Conference on Unmanned Aircraft Systems (ICUAS), 2015, Denver, CO, USA, pp. 236-245. DOI:10.1109/ICUAS.2015.7152296

3. Enright W.H. Continuous numerical methods for ODEs with defect control // Journal of Computational and Applied Mathematics, 2000, vol. 125 (1-2), pp. 159-170. DOI: 10.1016/S0377-0427(00)00466-0

4. Petukhov V.G. Method of continuation for optimization of interplanetary low-thrust trajectories // Cosmic Research, 2012, vol. 50 (3), pp. 249-261. DOI: 10.1134/S0010952512030069

5. Fountoulakis E., Paschos G.S., Pappas N. UAV Trajectory Optimization for Time Constrained Applications // IEEE Networking Letters, 2020, vol. 2 (3), pp. 136-139. DOI: 10.1109/LNET.2020.3007310

6. Gong J., Xie R. Trajectory optimization of unmanned aerial vehicle in vertical profile // In International Conference on Intelligent Robotics and Applications. Springer, Berlin, Heidelberg, 2012, pp. 404-410.

7. Sun S., Fan G., Fan Y. Trajectory Optimization of Unmanned Aerial Vehicle's Ascending Phase based on hp Adaptive Pseudospectral Method // In 3rd International Conference on Computer Engineering, Information Science & Application Technology (ICCIA 2019), Atlantis Press, 2019, pp. 523-533. DOI: 10.2991/iccia-19.2019.82

8. Visintini A., Perera T.D.P., Jayakody D.N.K. 3-D Trajectory Optimization for Fixed-Wing UAV-Enabled Wireless Network // IEEE Access, 2021, vol. 9, pp. 35045-35056. DOI: 10.1109/ACCESS.2021.3061163

9. Cuong N.D. Modeling and simulating the motion of autonomous flying devices, People’s Army Publisher, Hanoi, 2002.

10. Dien N. N., Van Toan N. Optimization of UAV landing taking into consideration of limitation on control based on solution of the boundary value problem by the parameter continuation // In Journal of Physics: Conference Series, IOP Publishing, 2019, vol. 1172, no. 1. DOI: 10.1088/1742-6596/1172/1/012075

11. Fluent Inc. Ansys Fluent 15 Users Guide, 2013. URL: https://vdocument.in/ansys-fluent-tutorial-guide-r-15.html?page=1

12. Александров А.А. Оптимальное управление летательным аппаратом с учётом ограничений на управление: автореферат дисс. канд. техн. наук. - Санкт Петербург, 2009, 134 с.

13. Kabanov S.A., Aleksandrov A.A. Applied optimal control problems, Saint Petersburg, Baltic State Technical University, 2007.

14. Dikusar V.V., Koska M., Figura A. A Parameter Extension Method for Solving Boundary Value Problems in Optimal Control Theory // Differential Equations, 2001, vol. 37 (4), pp. 479-484. DOI:10.1023/A:1019271321833

15. Петухов В.Г. Применение угловой независимой переменной и ее регуляризирующего преобразования в задачах оптимизации траекторий с малой тягой // Космические исследования. 2019. Т. 57, № 5. С. 373-385. DOI: 10.1134/S0023420619050066

16. Лёб Х.В., Могулкин А.И., Обухов В.А., Петухов В.Г. Анализ многоразового лунного транспортного корабля, использующего ядерную энергетическую установку // Труды МАИ. 2013. № 70. URL: http://trudymai.ru/published.php?ID=44497

17. Konstantinov M.S., Petukhov V.G., Leb Kh.V. Application of RIT-22 Thruster for InterhelioProbe Mission // Trudy MAI, 2012, no. 60. URL: https://trudymai.ru/published.php?ID=35420

18. Лёб Х.В., Петухов В.Г., Попов Г.А. Гелиоцентрические траектории космического аппарата с ионными двигателями для исследования Солнца // Труды МАИ. 2011. № 42. URL: http://trudymai.ru/published.php?ID=24275

19. Kuznetsov E.B. Optimal parametrization in numerical construction of curve // Journal of the Franklin Institute, 2007, vol. 344, pp. 658-671. DOI:10.1016/j.jfranklin.2006.02.036

20. Hairer E., Wanner G. Solving Ordinary Differential Equations I: Nonstiff Problems. Springer-Verlag, Berlin, 1993. DOI: 10.1007/978-3-540-78862-1

References

1. Horla D., Cieślak J. On obtaining energy-optimal trajectories for landing of UAVs, *Energies*, 2020, vol. 18 (8), pp. 2062. DOI:10.3390/en13082062

2. Rucco A., Aguiar A.P., Hauser J. Trajectory optimization for constrained UAVs: A virtual target vehicle approach, *In 2015 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2015, Denver, CO, USA, pp. 236-245. DOI:10.1109/ICUAS.2015.7152296

3. Enright W.H. Continuous numerical methods for ODEs with defect control, *Journal of Computational and Applied Mathematics*, 2000, vol. 125 (1-2), pp. 159-170. DOI: 10.1016/S0377-0427(00)00466-0

4. Petukhov V.G. Method of continuation for optimization of interplanetary low-thrust trajectories, *Cosmic Research,* 2012, vol. 50 (3), pp. 249-261. DOI: 10.1134/S0010952512030069

5. Fountoulakis E., Paschos G.S., Pappas N. UAV Trajectory Optimization for Time Constrained Applications, *IEEE Networking Letters*, 2020, vol. 2 (3), pp. 136-139. DOI: 10.1109/LNET.2020.3007310

6. Gong J., Xie R. Trajectory optimization of unmanned aerial vehicle in vertical profile, *In International Conference on Intelligent Robotics and Applications*. Springer, Berlin, Heidelberg, 2012, pp. 404-410.

7. Sun S., Fan G., Fan Y. Trajectory Optimization of Unmanned Aerial Vehicle's Ascending Phase based on hp Adaptive Pseudospectral Method, *In 3rd International Conference on Computer Engineering, Information Science & Application Technology (ICCIA 2019)*, Atlantis Press, 2019, pp. 523-533. DOI: 10.2991/iccia-19.2019.82

8. Visintini A., Perera T.D.P., Jayakody D.N.K. 3-D Trajectory Optimization for Fixed-Wing UAV-Enabled Wireless Network, *IEEE Access,* 2021, vol. 9, pp. 35045-35056. DOI: 10.1109/ACCESS.2021.3061163

9. Cuong N.D. *Modeling and simulating the motion of autonomous flying devices*, People’s Army Publisher, Hanoi, 2002.

10. Dien N. N., Van Toan N. Optimization of UAV landing taking into consideration of limitation on control based on solution of the boundary value problem by the parameter continuation, *In Journal of Physics: Conference Series*, IOP Publishing, 2019, vol. 1172, no. 1. DOI: 10.1088/1742-6596/1172/1/012075

11. *Fluent Inc. Ansys Fluent 15 Users Guide*, 2013. URL: https://vdocument.in/ansys-fluent-tutorial-guide-r-15.html?page=1

12. Aleksandrov A.A. *Optimal'noe upravlenie letatel'nym apparatom s uchetom ogranichenii na upravlenie* (Optimal control of aircraft, taking into account restrictions on control), dissertation abstract, Saint Petersburg, 2009, 134 p.

13. Kabanov S.A., Aleksandrov A.A. *Applied optimal control problems*, Saint Petersburg, Baltic State Technical University, 2007.

14. Dikusar V.V., Koska M., Figura A. A Parameter Extension Method for Solving Boundary Value Problems in Optimal Control Theory, *Differential Equations*, 2001, vol. 37 (4), pp. 479-484. DOI:10.1023/A:1019271321833

15. Petukhov V.G. *Kosmicheskie issledovaniya*, 2019, vol. 57, no 5, pp. 373-385. DOI: 10.1134/S0023420619050066

16. Leb Kh.V., Mogulkin A.I., Obukhov V.A., Petukhov V.G. *Trudy MAI*, 2013, no. 70. URL: http://trudymai.ru/eng/published.php?ID=44497

17. Konstantinov M.S., Petukhov V.G., Leb Kh.V. *Trudy MAI*, 2012, no. 60. URL: https://trudymai.ru/eng/published.php?ID=35420

18. Leb Kh.V., Petukhov V.G., Popov G.A. *Trudy MAI*, 2011, no. 42. URL: http://trudymai.ru/eng/published.php?ID=24275

19. Kuznetsov E.B. Optimal parametrization in numerical construction of curve, *Journal of the Franklin Institute*, 2007, vol. 344, pp. 658-671. DOI: 10.1016/j.jfranklin.2006.02.036

20. Hairer E., Wanner G. *Solving Ordinary Differential Equations I: Nonstiff Problems*. Springer-Verlag, Berlin, 1993. DOI: 10.1007/978-3-540-78862-1

Статья поступила в редакцию 22.05.2023

Одобрена после рецензирования 25.05.2023

Принята к публикации 27.06.2023

The article was submitted on 22.05.2023; approved after reviewing on 25.05.2023; accepted for publication on 27.06.2023