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Elements of mathematical phenomenology: II. Phenomenological approximate mappings

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Abstract

Mihailo Petrović (1868-1943) is an important Serbian mathematician and in general scientist, one of three Poincare's doctoral students. Paper starts with short citation of *Element of Mathematical Phenomenology* and *Phenomenological Mappings* published in Petrović's theory. Some of basic elements of mathematical phenomenology as it is elements of non-linear-functional transformations of coordinates from one to other functional curvilinear coordinate system and are presented in our previous published paper. Structural analogy between multi deformable system ais presented. Phenomenological approximate mappings on nonlinear phenomena, in local area around stationary points or stationary states, are presented. For obtaining approximate differential equations and approximate solutions in local area around singular points, linear and non-liner approximations are used. Method of local analysis based on phenomenological approximate mappings between local linear as well as nonlinear phenomena is power to obtain information of all local nonlinear phenomena in the nonlinear dynamics of the system for completing kinetic elements for global analysis of the system nonlinear dynamics and stability and to use different analogies.

Keywords: Mihailo Petrović (1868-1943), mathematical phenomenology; structural analogy, phenomenological approximate mappings; fractional order multi-deformable body system; fractional order modes; non-linear phenomena; local approximation; singular points; analogies; theorem**.**

I. Introduction

Mihailo Petrović (1868-1943) was an important Serbian mathematician and scienist, one of three Poincaré's doctoral students. His professors were world important scientists, such as Poincaré (Jules Henri Poincaré (1854 –1912)), Appall (Paul Appell (1855 –1930)), Hermite (Charles Hermite (1822 –1901)), Picard (Charles Émile Picard (1856 –1941)), Painlevé (Paul Painlevé (1863 –1933)), Bousinesq (Joseph Valentin Boussinesq (1842 –1929)) and others. He published the books "*Elements of Mathematical Phenomenology"* [34] in 1911 and "*Phenomenological Mapping"* [35] in 1933.

A short presentation of this book was in French, titled by *Mecanismes communs aux phenomens disparates*, Paris 1921 [36].

Using elements of mathematical phenomenology and in particular different types of analogies, and qualitative, structural and mathematical elements, it is possible to make precise or approximate phenomenological mappings of phenomena [35] from global to local area of system kinetic parameters, but is necessary to add corresponding conditions of restrictions. Also, it is possible to make analogy between two different phenomena in two or more systems from disparate areas of science (see references [3-4] and [26]) and identify equal or similar properties expressed by elements of mathematical phenomenology.

II. STRUCTURAL ANALOGY

II.1. INTRODUCTION-COMMENTS

Let starting with rigid body motion. For moving a rigid body from one position to other it is possible to produce by one translation and one pure rotation around a pole.

A deformation of elementary part around a point of deformable body is produced by extension of the line elements drown from one point and by rotation of these line elements. And, in results is produced a change of angles between line elements and in final results are two deformations: first component - pure change of dimensions of deformable body with change volume of deformable body, but no change form and second component - pure deformation of deformable body form without change volume of deformable body. These deformations are dilatations and sliding (deformation by line element rotations). For details see middle column in Table 2. from our Reference [2]).

Let compare deformation of tangent space of a vector position of a kinetic point in curvilinear orthogonal coordinates during motion of this point. In third part of Reference [2] we present that motion of a kinetic point tangent space of their vector position, in curvilinear orthogonal coordinate system, produce a pure rotation around fixed or moving axes with corresponding angular component velocities followed by deformation of tangent space as it is with element of deformable body. In this case when curvilinear coordinates are orthogonal no pure deformation of form of tangent space, only change of volume of tangent space. (See elements of mathematical phenomenology in Table 1 and Table 2. ig

Reference [2]). In the case that curvilinear coordinate system is no orthogonal and change s of angles between curvilinear coordinate line appear, when kinetic point moves, two component of the pure deformation change of the volume and change of the form of this tangent space are produced. Between strain state of deformed body elementary part in dynamical sates and deformation of a elementary volume of tangent space of a moving kinetic point vector position in discrete configuration, a qualitative analogy is possible to identified. This analogy is between strain state of deformable body and strain sate of tangent space in curvilinear coordinate system by translation and rotation as in the case of rigid body change position and pure deformation extension of line elements in three directions with extension of coordinate lines as contour of tangent space element of vector position of a kinetic point and change of the angles between these coordinate lines or lines elements.

In previous description is possible to talk about elements of mathematical phenomenology and in pure geometry-kinematical system and also by qualitative analogy about deformation an elementary element around a point in the deformed deformable body. By presented analysis it is visible a structural analogy between mechanisms of deformable body deformation and deformation of a tangent space of a moving kinetic point position vector (see References [2], [22], [37] and [38]).

This structural analogy between two mechanisms of deformation is in translation and rotation as well as deformation by extension (correspond to translation) and sliding (correspond to rotation).

Structural analogies are visible in different scale in building different fractals by iterative dynamics (Julia sets, Maldenbrot sets and other).

II.2. STRUCTURAL ANALOGIES BETWEEN EIGEN TIME FUNCTIONS OF TRANSVERSAL VIBRATIONS OF MULTI- DEFORMABLE BODY SYSTEMS

Let consider transversal vibrations of multi-deformable-body system, containing coupled, with same contour and boundary conditions, a finite number of deformable bodies, coupled by pure elastic, linear or nonlinear, viscoe-elastic or hereditary or fractional order discrete continuum layers distributed between deformable bodies (beams, or plates, or membranes, or belts). Discrete continuum layers contain distributed standard light elements with corresponding type of constitutive relations (*for detail see References* [9-21], [23-24] and [28-30]). For our consideration of the elements of mathematical phenomenology, and, in particular, of structural analogies, for first we need to analyze physical structure of models of multi-body system [14]. After that, as second it is necessary to list partial differential equations of transversal vibrations of multi-deformable bodies coupled by discrete continuum layers containing homogeneous distributed standard light fractional order elements. Use multi-body systems, presented in Figure 1, by two way of the analysis is possible to indicate more than one type of structural analogy as well as other types of analogies, mathematical and qualitative analogies [11].

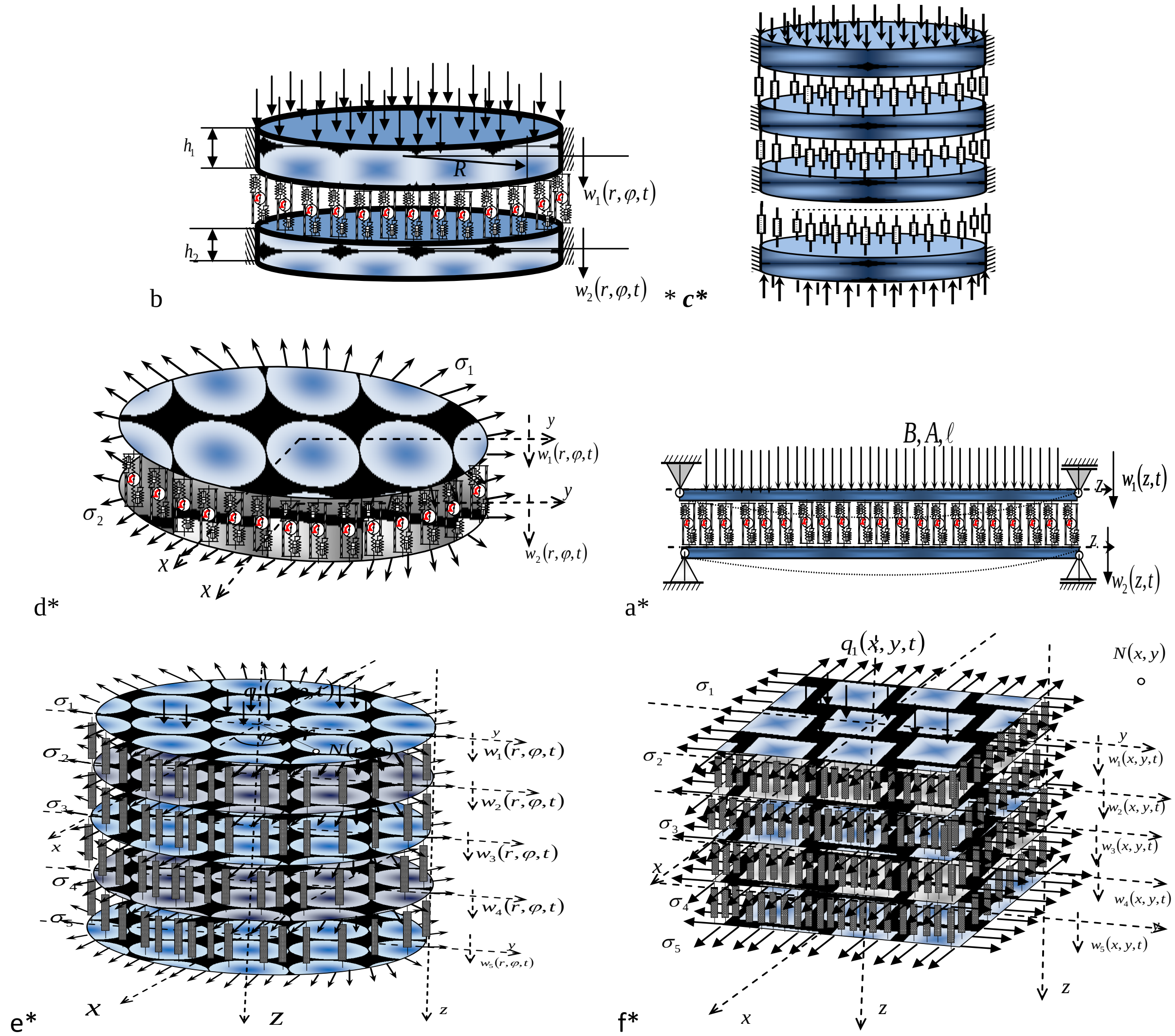


Figure 1. Models of multi deformable body system: a* two beam visco-elastic system, with beams same length and boundary conditions; b* and c* two plate and four plate nonlinear dynamical system, with plates same contours and boundary conditions; d* two membrane visco-elastic system, with membranes same contours and boundary conditions; e* five circular membrane fractional order systems and f* five rectangular membrane fractional order hybrid system, with membranes same contours and boundary conditions.

In Figure 1, a numbers of models of multi deformable body system are presented. Two beam visco-elastic system presented in Figure 1.a*, with beams [14], same length and boundary conditions, are couples by discrete continuum visco-elastic layers 1] In Figure 1.b* and c* are visible two plates [28-30] nd a four plate [11] nonlinear dynamical system, with plates same contours and boundary conditions, coupled by discrete continuum layers consists of distributed standard light visco-elastic elements. Two membrane visco-elastic system, with ideal elastic membranes, same contours and boundary conditions, and coupled by discrete light visco-elastic layers is presented in Figure 1. d. Five circular membrane fractional order systems and five rectangular membrane fractional order hybrid system, with ideal elastic membranes, same contours and boundary conditions, coupled by standard light fractional order elements distributed between membranes are presented in Figures 1. e* and f*. After analysis, it is possible by intuition, without mathematical descriptions indicate present qualitative and structural analogies between these all presented multi-body system dynamics [11]. Let's start with analysis of displacements in transversal directions for simple models, and neglecting other displacements in axial and lateral directions of beams, plate or membranes. Forces of interactions accept that are in transversal directions for simplest models. The system of partial fractional order differential equations are listed (see References [14] and in details [11]).

a* Governing partial fractional order differential equations of a hybrid multi deformable (three) beam system transversal oscillations on a discrete continuum fracional order layer in which beams are coupled by discrete continuum fractional order layers, are (see References by Rašković [37-381] and by Hedrih (Stevanović)[11]):

$$\rho_1 A_1 \frac{\partial^2 w_1(x,t)}{\partial t^2} = -\mathsf{B}_1 \frac{\partial^4 w_1(x,t)}{\partial x^4} + c_{0(1.2)}\left[w_2(x,t) - w_1(x,t)\right] + c_{\alpha(1,2)}\mathsf{D}_t^{\alpha}\left[w_2(x,t) - w_1(x,t)\right] + q_1(x,t)$$

$$\begin{aligned}\rho_2 A_2 \frac{\partial^2 w_2(x,t)}{\partial t^2} = -\mathsf{B}_2 \frac{\partial^4 w_2(x,t)}{\partial x^4} - c_{0(1.2)}\left[w_2(x,t) - w_1(x,t)\right] - c_{\alpha(1,2)}\mathsf{D}_t^{\alpha}\left[w_2(x,t) - w_1(x,t)\right] + \\ + c_{0(2.3)}\left[w_3(x,t) - w_2(x,t)\right] + c_{\alpha(2,3)}\mathsf{D}_t^{\alpha}\left[w_3(x,t) - w_2(x,t)\right] + q_2(x,t)\end{aligned} \tag{1}$$

$$\rho_3 A_3 \frac{\partial^2 w_3((x,t)}{\partial t^2} = -\mathrm{B}_3 \frac{\partial^4 w_3(x,t)}{\partial x^4} - c_{0(2.3)}[w_3(x,t) - w_2(x,t)] - c_{\alpha(2,3)} \mathrm{D}_t^\alpha [w_3(x,t) - w_2(x,t)] -$$
$$+ c_{0(1.2)}[-w_3(x,t)] + c_{\alpha(3,4)} \mathrm{D}_t^\alpha [-w_3(x,t)] + q_3(x,y)$$

where $\mathrm{B}_k = E_k I_{zk}$, $k = 1,2,3$ are flexural rigidity of the beams.

For complete description of previous hybrid fractional order system it is necessary to define and add crresponding baundary conditions and corresponding initial conditions. All three beams are same length and same boundary conditions. This fact permit us to suppose that eigen amplitude functions for all three beams is possible to take in the same form $\mathbf{W}_n(x)$, $n = 1,2,3,4...\infty$ and solutions suppose in the following forms:

$$w_k(x,t) = \sum_{n=1}^{\infty} W_n(x) T_{k(n)}(t), \; k = 1,2,3 \tag{2}$$

and that distributed excitation along beams lengths are :

$$\frac{q_k(x,t)}{\rho_k} = \sum_{m=1}^{M} h_{0k,n} W_n(x) \sin(\Omega_{kn} t + \vartheta_{k,n}), k = 1,2,3 \tag{3}$$

For presenting structural analogy, we take that external excitations along bems are equal to zero, and without losing generality, it is possible to obtain three independent eigen main fractional order modes of free vibrations for each eigen time function $T_{k(n)}(t)$, $k = 1,2,3$, $n = 1,2,3,....,\infty$ corresponding to one of the eigen amplitude function $\mathbf{W}_n(x)$, $n = 1,2,3,4...\infty$ from infinite set of eigen amplitude described by independent eigen main coordinates $\xi_{(n)s}$, $n = 1,2,3,....,\infty, s = 1,2,3$ for free vibrations in the following form:

$$\ddot{\xi}_{(n)s}(t) + \tilde{\omega}_{n(s)}^2 \xi_{(n)s}(t) + \tilde{\omega}_{\alpha(n)(s)}^2 \mathrm{D}_t^\alpha [\xi_{(n)s}(t)] = 0, \qquad n = 1,2,3,....,\infty, s = 1,2,3 \tag{4}$$

(for details see References [27]).

Also, for multi (three) beam fractional order system forced vibrations, it is possible to obtain three independent forced eigen main fractional order modes of forced vibrations for each of time function $T_{k(n)}(t)$, $k=1,2,3$, $n=1,2,3,....,\infty$ corresponding to one of the eigen amplitude function $\mathbf{W}_n(x)$, $n=1,2,3,4...\infty$ from infinite set of eigen amplitude described by independent eigen main coordinates $\xi_{(m)s}$, $n=1,2,3,....,\infty, s=1,2,3$ for forced vibrations. Solution of the fractional order differential equations (4) is known see References by Rašković [37-381] and by Hedrih (Stevanović)[11]).

b* Governing partial fractional order differential equations of a hybrid multi deformable (three) plate fractional order system transversal oscillations on a discrete continuum fractional order layer in which plates are coupled by discrete continuum fractional order layers, are (see References by Rašković [37-381] and by Hedrih (Stevanović) [11] and [`6]):

$$\rho_1 h_1 \frac{\partial^2 w_1(x,y,t)}{\partial t^2} = -\mathsf{D}_1 \Delta\Delta w_1(x,y,t) + c_{0(1.2)}[w_2(x,y,t) - w_1(x,y,t)] + c_{\alpha(1,2)}\mathsf{D}_t^{\alpha}[w_2(x,y,t) - w_1(x,y,t)] + q_1(x,y,t)$$

$$\begin{aligned} \rho_2 A_2 \frac{\partial^2 w_2(x,y,t)}{\partial t^2} &= -\mathsf{D}_2 \Delta\Delta w_2(x,y,t) - c_{0(1.2)}[w_2(x,y,t) - w_1(x,y,t)] - c_{\alpha(1,2)}\mathsf{D}_t^{\alpha}[w_2(x,y,t) - w_1(x,y,t)] + \\ &+ c_{0(2.3)}[w_3(x,t) - w_2(x,t)] + c_{\alpha(2,3)}\mathsf{D}_t^{\alpha}[w_3(x,t) - w_2(x,t)] + q_2(x,y,t) \end{aligned} \quad (5)$$

$$\begin{aligned} \rho_3 A_3 \frac{\partial^2 w_3(x,y,t)}{\partial t^2} &= -\mathsf{D}_3 \Delta\Delta w_3(x,y,t) - c_{0(2.3)}[w_3(x,y,t) - w_2(x,y,t)] - c_{\alpha(2,3)}\mathsf{D}_t^{\alpha}[w_3(x,y,t) - w_2(x,y,t)] + \\ &+ c_{0(1.2)}[w_4(x,y,t) - w_3(x,y,t)] + c_{\alpha(3,4)}\mathsf{D}_t^{\alpha}[w_4(x,y,t) - w_3(x,y,t)] + q_3(x,y,t) \end{aligned}$$

where D_k , $k=1,2,3$ flexural rigidity of the plates.

For complete description of previous hybrid fractional order system it is necessary to define and add crresponding baundary conditions and corresponding initial conditions. All three plates are same boundary contours and same boundary conditions. This fact permit us to suppose that eigen amplitude functions for all three plates is possible to take in the same form $W_{nm}(x,y)$, $n,m=1,2,3,4..\infty$ and solutions suppose in the following forms:

$$w_k(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{nm}(x,y)T_{k(nm)}(t),\ k=1,2,3 \tag{6}$$

and that distributed transversal excitation along plate middle surface s are :

$$\frac{q_k(x,y,t)}{\rho}=\sum_{m=1}^{M}\sum_{n=1}^{N}h_{0k,nm}W_{nm}(x,y)\sin(\Omega_{knm}t+\vartheta_{k,nm}),\qquad k=1,2,3 \tag{7}$$

For presenting structural analogy we take that external excitations along plates are equal to zero, and without losing generality, it is possible to obtain three independent eigen main fractional order modes of free vibrations for each time function $T_{k(nnn)}(t)$, $k=1,2,3$, $n,m=1,2,3,4..\infty$ corresponding to one of the eigen amplitude function $W_{nm}(x,y)$, $n,m=1,2,3,4..\infty$ from infinite set of eigen amplitude described by independent eigen main coordinates $\xi_{(nm)s}$, $n,m=1,2,3,....,\infty, s=1,2,3$ for free vibrations in the following form:

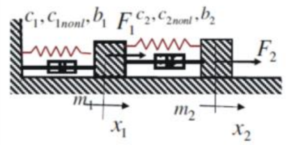
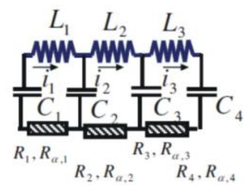
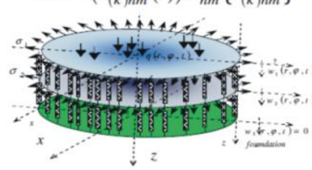
$$\ddot{\xi}_{(nm)s}(t)+\tilde{\omega}^2_{nm(s)}\xi_{(nm)s}(t)+\tilde{\omega}^2_{\alpha(nm)(s)}\mathsf{D}_t^{\alpha}\left[\xi_{(nm)s}(t)\right]=0,\qquad n,m=1,2,3,....,\infty, s=1,2,3 \tag{8}$$

Formula transformation between egen time functions and eigen main fractional order coordinate are:

$$T_{k(nm)}(t)=\sum_{s=1}^{s=3}\mathbf{K}^{(s)}_{(nm)3k}\xi_{(nm)s}(t),\qquad k=1,2,3\,,n,m=1,2,3,....,\infty \tag{9}$$

analogous as for corresponding linear system expressions for coordinate transformations.

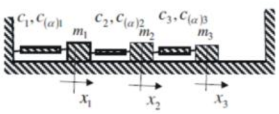
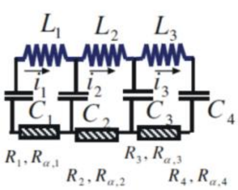
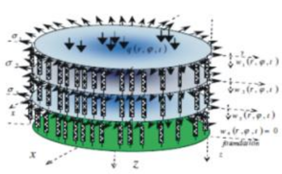
Table 1. Qualitative and mathematical analogous nonlinear system energies: a* mechanical nonlinear system and b* electrical nonlinear system with two degree of free4dom and c* eigen time functions in one eigen amplitude form of double membrane oscillations coupled by nonlinear discrete continuum dissipative layers (approximative).

	Kinetic energy	Potential energy	Rayleigh function of ergy dissipation	Energy relations in frational order system
a*	For mass particles $2\mathbf{E}_k=(\dot{x})\mathbf{A}\{\dot{x}\}$ $(x)=(x_1 \quad x_2)$ $2\mathbf{E}_k=\sum_{s=1}^{s=2}\dot{\eta}_s^2$ η_s, $s=1,2$ eigen normal coordinates (modes) of displacements	For linear elastic springs $2\mathbf{E}_p=(x)C\{x\}+$ $+\frac{1}{2}c_{1nonlin}x_1^4+\frac{1}{2}c_{2nonlin}(x_2-x_1)^4$ $2\mathbf{E}_p=\sum_{s=1}^{s=2}\omega_s^2\eta_s^2+\mathbf{E}_{pnonlin}$ $\ddot{\xi}_s+\omega_s^2\xi_s+f_s(\xi_s,\xi_l)=0$ $s,k=1,2; \quad k\neq s$	Power of linear dissipative forces in standard damping elements $2\Phi=(\dot{x})\mathbf{B}\{\dot{x}\}$ 	System total mechanical energy rate $\frac{d\mathbf{E}}{dt}=-2\Phi$, Eigen main $for \quad \alpha\neq 0, 0<\alpha<1$
b*	For inductors $2\mathbf{E}_k=(\dot{q})\mathbf{A}\{\dot{q}\}$ $(q)=(q_1 \quad q_2)$, $\dot{q}=i$ $2\mathbf{E}_k=\sum_{s=1}^{s=2}\dot{\eta}_s^2$ η_s, $s=1,2$ eigen normal coordinates (modes) of electricity	For linear capacitors $2\mathbf{E}_p=(q)C\{q\}+$ $+\frac{1}{2}c_{1nonlin}q_1^4+\frac{1}{2}c_{2nonlin}(q_2-q_1)^4$ $2\mathbf{E}_p=\sum_{s=1}^{s=2}\omega_s^2\eta_s^2+\mathbf{E}_{pnonlin}$ $\ddot{\xi}_s+\omega_s^2\xi_s+f_s(\xi_s,\xi_l)=0$ $s,k=1,2; \quad k\neq s$	Power of dissipative electrical voltage in standard linear resistors $2\Phi=(\dot{q})\mathbf{B}\{\dot{q}\}$ 	System total electrical energy rate $\frac{d\mathbf{E}}{dt}=-2\Phi$
c*	$\mathbf{A}_{nm}=(a_{nm,kj})_{\to\, j=1,2}^{\downarrow\, k=1,2}$ $(T_{(k)nm}(t))=(T_{(1)nm}(t) \quad T_{(2)nm}(t))$ $2\mathbf{E}_{K,nm}=(\dot{T}_{(k)nm}(t))\mathbf{A}_{nm}\{\dot{T}_{(k)nm}\}$ $2\mathbf{E}_{K,nm}=\sum_{s=1}^{s=r}\tilde{a}_{nm,ss}\dot{\xi}_{nm,s}^2=\sum_{s=1}^{s=r}2\mathbf{E}_{K,nm,s}$ $n,m=1,2,3,....,\infty$	Approximate phenomenological mappings $\mathbf{C}_{nm}=(c_{nm,kj})_{\to\, j=1,2}^{\downarrow\, k=1,2}$ $2\mathbf{E}_p=(T_{(k)nm})\mathbf{C}_{nm}\{T_{(k)nm}\}+$ $+\frac{1}{2}c_{1,nm,nonlin}T_{(1)nm}^4+$ $+\frac{1}{2}c_{2,nm,nonlin}(T_{(2)nm}-T_{1nm})^4$ $2\mathbf{E}_p=\sum_{s=1}^{s=2}\omega_s^2\eta_s^2+\mathbf{E}_{pnonlin}$ $\ddot{\xi}_s+\omega_s^2\xi_s+f_s(\xi_s,\xi_l)=0$ $s,k=1,2; \quad k\neq s$ $n,m=1,2,3,....,\infty$	Aproximative value of power of linear dissipative forces in standard damping elements correspond to one eigen amplitude mode $2\Phi=(\dot{T}_{(k)nm}(t))\mathbf{B}_{nm}\{\dot{T}_{(k)nm}\}$ 	$\frac{d\mathbf{E}_{nm}}{dt}=-2\Phi_{nm}(\dot{T}_{nm}(t))(t)$

Also, for multi (three) plate fractional order system forced vibrations, it is possible to obtain three independent eigen main fractional order modes of forced vibrations for each eigen time function $T_{k(nnn)}(t)$, $k=1,2,3$, $n,m=1,2,3,4..\infty$ corresponding to one of the eigen amplitude function $W_{nm}(x,y)$, $n,m=1,2,3,4..\infty$ from infinite set of eigen amplitude described by independent eigen main coordinates $\xi_{(nm)s}$, $n,m=1,2,3,....,\infty, s=1,2,3$ for forced vibrations.

Table 2. Qualitative and mathematical analogous fractional order system energies: a* mechanical fractional order system and b* electrical fractional order system and c* * eigen time functions in one

eigen amplitude form of double membrane oscillations coupled by fractional order discrete continuum dissipative layers.

<table>
<tr><th></th><th>Kinetic energy</th><th>Potential energy</th><th>Generalized function of fracrional order energy dissipation</th><th>Energy relations in frational order system</th></tr>
<tr><td>a*</td><td>For mass particles
$2\mathbf{E}_k = (\dot{x})\mathbf{A}\{\dot{x}\}$
$2\mathbf{E}_k = \sum_{s=1}^{s=n} \dot{\eta}_s^2$
η_s, $s = 1,2,3,...,n$ eigen normal coordinates (modes) of displacements

</td><td>For linear elastic springs
$2\mathbf{E}_p = (x)C\{x\}$
$2\mathbf{E}_p = \sum_{s=1}^{s=n} \omega_s^2 \eta_s^2$
$\mathbf{A}\{\ddot{x}\} + C_\alpha\{\mathfrak{D}_t^\alpha\{x\}\} + C\{x\} = \{0\}$
$\ddot{\xi}_s + \omega_{(\alpha)s}^2 \mathfrak{D}_t^\alpha\{\xi_s\} + \omega_s^2 \xi_s = 0$
$0 \le \alpha \le 1$, $s = 1,2,3,...,n$</td><td>Power of fractional order dissipative forces in standard fractional order elements
$2\mathbf{P}_{\alpha\neq 0} = (\mathfrak{D}_t^\alpha\{x\})C_\alpha\{\mathfrak{D}_t^\alpha\{x\}\}$,
$for \quad \alpha \neq 0, 0 < \alpha < 1$
$2\mathbf{P}_\alpha = \sum_{s=1}^{s=n} \omega_{(\alpha),s}^2 (\mathfrak{D}_t^\alpha[\eta_s])^2$
$for \quad \alpha \neq 0, 0 < \alpha < 1$
$P(t) = -\{c_0 x(t) + c_\alpha \mathfrak{D}_t^\alpha[x(t)]\}$</td><td>System total mechanical energy rate
$\frac{d\mathbf{E}}{dt} = -2\Phi - \sum_{k=1}^{k=n}\sum_{j=1}^{j=n} \dot{x}_k \frac{\partial \mathbf{P}_\alpha}{\partial(\mathfrak{D}_t^\alpha[x_j])}$, Eig
$for \quad \alpha \neq 0, 0 < \alpha < 1$
en main fractional order independent mode total mechanical energy rate
$\frac{d\mathbf{E}_s}{dt} = -2\Phi_s - \dot{\xi}_{ss} \frac{\partial \mathbf{P}_\alpha}{\partial(\mathfrak{D}_t^\alpha[\xi_s])}$,
$for \quad \alpha \neq 0, 0 < \alpha < 1, s = 1,2,3...,n$</td></tr>
<tr><td>b*</td><td>For inductors
$2\mathbf{E}_k = (\dot{q})\mathbf{L}\{\dot{q}\}$
$2\mathbf{E}_k = \sum_{s=1}^{s=n} \dot{\eta}_s^2$
η_s. $s = 1,2,3,...,n$ eigen normal coordinates (modes) of electricity

</td><td>For linear capacitors
$2\mathbf{E}_p = (q)\mathbf{C}*\{q\}$
$2\mathbf{E}_p = \sum_{s=1}^{s=n} \omega_s^2 \eta_s^2$
$\mathbf{A}\{\ddot{x}\} + C_\alpha\{\mathfrak{D}_t^\alpha\{x\}\} + C\{x\} = \{0\}$
$\ddot{\xi}_s + \omega_{(\alpha)s}^2 \mathfrak{D}_t^\alpha\{\xi_s\} + \omega_s^2 \xi_s = 0$
$0 \le \alpha \le 1$, $s = 1,2,3,...,n$</td><td>Power of fractional order dissipative electrical voltage in standard fractional order resistors
$2\mathbf{P}_{\alpha\neq 0} = (\mathfrak{D}_t^\alpha\{q\})\mathbf{R}_\alpha\{\mathfrak{D}_t^\alpha\{q\}\}$,
$for \quad \alpha \neq 0, 0 < \alpha < 1$
$2\mathbf{P}_\alpha = \sum_{s=1}^{s=n} \omega_{(\alpha),s}^2 (\mathfrak{D}_t^\alpha[\eta_s])^2$
$for \quad \alpha \neq 0, 0 < \alpha < 1$
$V(t) = -\left\{\frac{1}{C_0} q(t) + R_\alpha \mathfrak{D}_t^\alpha[q(t)]\right\}$</td><td>System total electrical energy rate
$\frac{d\mathbf{E}}{dt} = -2\Phi - \sum_{k=1}^{k=n}\sum_{j=1}^{j=n} \dot{x}_k \frac{\partial \mathbf{P}_\alpha}{\partial(\mathfrak{D}_t^\alpha[x_j])}$,
$for \quad \alpha \neq 0, 0 < \alpha < 1$
Eigen main fractional order independent mode total electrical energy rate
$\frac{d\mathbf{E}_s}{dt} = -2\Phi_s - \dot{\xi}_{ss} \frac{\partial \mathbf{P}_\alpha}{\partial(\mathfrak{D}_t^\alpha[\xi_s])}$,
$for \quad \alpha \neq 0, 0 < \alpha < 1, s = 1,2,3...,n$</td></tr>
<tr><td>c*</td><td>$\mathbf{A}_{nm} = (a_{nm,kj})_{\to\, j=1,2,3,...,r}^{\downarrow\, k=1,2,3,...,r}$
$2\mathbf{E}_{K,nm} = (\dot{T}_{(k)nm}(t))\mathbf{A}_{nm}\{\dot{T}_{(k)nm}\}$
$2\mathbf{E}_{K,nm} = \sum_{s=1}^{s=r} \tilde{a}_{nm,ss}\dot{\xi}_{nm,s}^2 = \sum_{s=1}^{s=r} 2\mathbf{E}_{K,nm,s}$
</td><td>$\mathbf{C}_{nm} = (c_{nm,kj})_{\to\, j=1,2,3,...,r}^{\downarrow\, k=1,2,3,...,r}$
$2\mathbf{E}_{p,nm} = (T_{(k)nm})\mathbf{C}_{nm}\{T_{(k)nm}\}$
$2\mathbf{E}_{p,nm} = \sum_{s=1}^{s=r} \tilde{c}_{nm,ss}\xi_{nm,s}^2 = \sum_{s=1}^{s=r} 2\mathbf{E}_{pot,nm,s}$</td><td>$\mathbf{C}_{\alpha,nm} = (c_{\alpha,nm,kj})_{\to\, j=1,2,3,...,r}^{\downarrow\, k=1,2,3,...,r}$
$2\mathbf{P}_{\alpha\neq 0,nm} = (\mathfrak{D}_t^\alpha\{T_{(k),nm}\})\mathbf{C}_{\alpha,nm}\{\mathfrak{D}_t^\alpha\{T_{(k),nm}\}\}$
$2\mathbf{P}_{\alpha\neq 0,nm} = (\mathfrak{D}_t^\alpha\{\xi_{nm,s}\})\mathfrak{C}_{\alpha,nm}\{\mathfrak{D}_t^\alpha\{\xi_{nm,s}\}\}$
$2\mathbf{P}_{\alpha\neq 0,nm} = \sum_{s=1}^{s=r} \tilde{c}_{(\alpha)nm,ss}(\mathfrak{D}_t^\alpha\{\xi_{nm,s}\})^2$
$2\mathbf{P}_{\alpha\neq 0,nm} = \sum_{s=1}^{s=r} 2\mathbf{P}_{\alpha\neq 0,nm,s} \quad for \quad \alpha \neq 0$</td><td>$\frac{d\mathbf{E}_{nm}}{dt} = -2\Phi_{nm}(\dot{T}_{nm}(t)) - \frac{\partial \mathbf{P}(\mathfrak{D}_t^\alpha[T_{nm}(t)])}{\partial\langle\mathfrak{D}_t^\alpha[T_{nm}(t)]\rangle}\dot{T}_{nm}(t)$
$\frac{d\mathbf{E}_{nm}}{dt} = -b_{nm}\langle\dot{T}_{nm}(t)\rangle^2 - b_{\alpha,nm}\mathfrak{D}_t^\alpha[T_{nm}(t)]\dot{T}_{nm}(t)$
$\frac{d\mathbf{E}_{nm,s}}{dt} = -2\Phi_{nm,s} - \dot{\xi}_{nm,s}\frac{\partial \mathbf{P}_{\alpha,nm}}{\partial(\mathfrak{D}_t^\alpha[\xi_{nm,s}])}$,
$for \quad \alpha \neq 0, 0 < \alpha < 1, s = 1,2,3...,n$</td></tr>
</table>

c*Governing partial fractional order differential equations of a hybrid multi (three) deformable membrane fractional order system transversal oscillations on a discrete continuum fractional order layer in which plates are coupled by discrete continuum fractional order layers are analogous as in case b*. (see References by Rašković [37-381] and by Hedrih (Stevanović) [11] and [24]).

Comparing listed mathematical descriptions for multi beam system, multi plare system and multi membrane system, and obtained results in the form of independent three eigen main fractional order modes for each eigen time functions correspond to eigen amplitude functions, (2) and (6) it is valid to conclude about present mathematical analogy in the basis of the elements of mathematical phenomenology – similar systems of partial fractional differential equations with equal structure; each of eigen time functions is composed by tree eigen main fractional order independent time modes in each of three corresponding eigen amplitude modes.

Listed examples, are in the structural analogy in displacements, eigen time functions, eigen maun fractional order modes, (4) and (8), as weel as eigen main coordinates. This structural nalogy by inductions permit to made conclusions for the structural and mathematical analogies between multi-beam fractional order system transversal oscillations, and multi-plate fractional order system transversal oscillations and mlti-membrane fractional order system transversal vibrations with corresponding boundary conditions (*for detail see References* [9-21], [23-24] and [28-30]).

Next, it is possible to indicate analogy between chain fractional order discrete system and generalized coordinates and eigen time functions functions $T_{k(nnn)}(t)$, $n,m=1,2,3,4..\infty$ of multi-deformable fractional order system vibrations, and also between corresponding eigen main fractional order modes in these systems. Also, a model of

Double DNA helix chain mechanical model is analogous system with previous defined (*for detail see References* [9-21], [23-24] and [28-30]).

In Table 1. elements of qualitative and mathematical analogous nonlinear system energies: a* for mechanical nonlinear system and b* for electrical nonlinear system with two degree of freedom are presented, and also in approximate analogy c* for eigen time functions in one eigen amplitude form of double membrane oscillations coupled by non-linear discrete continuum dissipative layers are presented (see References [2],[5] and [7]).

In Table 2 elements of mathematical phenomenology for qualitative and mathematical analogous fractional order system energies: a* for mechanical fractional order system and b* for electrical fractional order system and c* for eigen time functions in one eigen amplitude form of double membrane oscillations coupled by fractional order discrete continuum dissipative layers are presented (see References [2], [5] and [7]).

III. PHENOMENOLOGICAL MAPPINGS

This chapter related to approximation of differential equations and approximation of solution around stationary singular point or around some stationary state or around known linear or nonlinear solution as a type of *phenomenological approximate mappings*, useful for investigation properties of free or forced regimes or stability of approximate system dynamics. Also, using phenomenological approximate mappings is possible to investigate stability or appearance singular regimes as it is resonance, dynamical absorption, bifurcation in local or global area or local area of some system parameters.

II.1. Linearization of nonlinear differential equations of a dynamics of the system around stationary point or stationary dynamics state is simplest phenomenological mapping. In the case that a nonlinear differential equation and nonlinear phenomena are substituted by approximate description, using linear differential equation for description of corresponding nonlinear by approximate linear phenomena it is valid only in a bounded area of kinetic parameters and in a short time interval of system non-linear dynamics.

Approximation of the solution of differential equation is also phenomenological approximate mapping [34-36] of a nonlinear differential equation solution into its approximation with corresponding restrictions of valid area of valid and acceptable applications. Also, when we take into account approximate differential equations and corresponding solution around all singularities of a complex system non-linear dynamics and, then obtained differential equations and approximation of the solutions present corresponding phenomenological approximate mappings, each around a singular state or singular point to corresponding simple linear, or simplest nonlinear oscillators, or fractional order oscillators or other type oscillator with one degree of freedom.

In Table 3, elements of phenomenological approximate mappings of nonlinear differential equations of different models of Watt's regulator non-linear dynamics around singular points is presented, examples of linearization or linear approximation around singular points of a trigger of coupled three singular points are presented. In same Table, phase portraits for three different models of Watt's regulator non-linear dynamics depending of

Table 3. Phenomenological approximate mappings of nonlinear differential equation and its approximate solution around singular points – Examples of linearization or linear approximation of a Watt's regulator nonlinear differential equation around singular points of a trigger of coupled three singular points

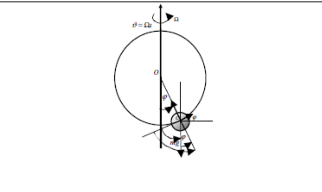
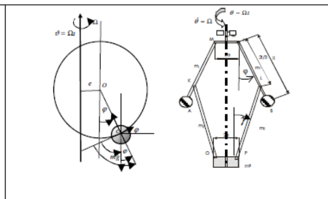
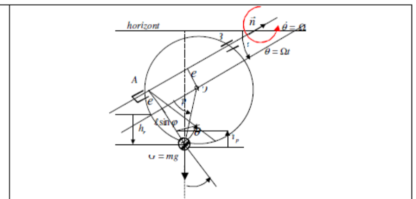
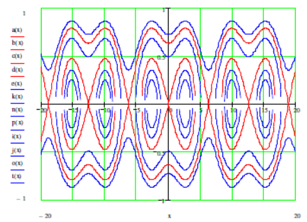
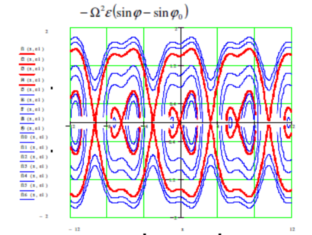
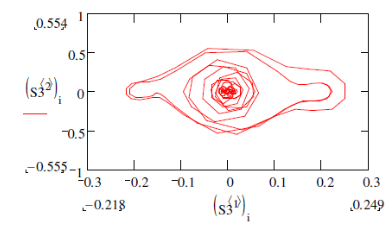
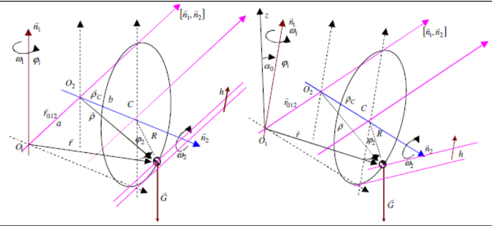
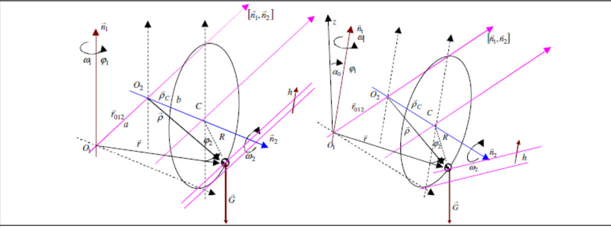
Dynamical model Parameters $\dot{\theta}=\Omega$, $\lambda=\frac{g\sin\alpha}{\ell\Omega^2}$, $\Omega_{rez}^2=\frac{g\sin\alpha}{2\ell}$, $\varepsilon=\frac{e}{\ell}$.			
Differential equation	$\ddot{\varphi}+\Omega^2(\lambda-\cos\varphi)\sin\varphi=0$	$\ddot{\varphi}+\Omega^2\langle\lambda-\cos\varphi\rangle\sin\varphi-\Omega^2\varepsilon\cos\varphi=0$	$\ddot{\varphi}+\Omega^2(\lambda-\cos\varphi)\sin\varphi=\Omega^2\lambda ctg\alpha\cos\varphi\cos\Omega t$ $\varepsilon=\frac{e}{\ell}=0$
Phase trajectory	$\dot{\varphi}^2=\dot{\varphi}_0^2-2\Omega^2\left\langle-\lambda(\cos\varphi-\cos\varphi_0)+\frac{1}{4}(\cos 2\varphi-\cos 2\varphi_0)\right\rangle$ 	$\dot{\varphi}^2=\dot{\varphi}_0^2-2\Omega^2\left\langle-\lambda(\cos\varphi-\cos\varphi_0)+\frac{1}{4}(\cos 2\varphi-\cos 2\varphi_0)\right\rangle$ $-\Omega^2\varepsilon(\sin\varphi-\sin\varphi_0)$ 	Numerical integration for obtaining a phase trajectory 
Singular points	$\varphi_s=s\pi$, $s=0,\pm1,\pm2,\pm3,....$ 1* $\lambda>1$ Stable centre type points $\varphi_0=0$, $\varphi_{s=2k}=2k\pi$, $s=0,\pm1,\pm2,\pm3,....$ No stable saddle points $\varphi_1=\pi$, $\varphi_{s=2k+1}=(2k+1)\pi$, $s=0,\pm1,\pm2,\pm3,....$ 2* $\lambda<1$ No stable saddle points $\varphi_s=s\pi$, $s=0,\pm1,\pm2,\pm3,....$ Stable centre type points $\varphi_s=\pm\arccos\lambda+2s\pi$, $s=0,\pm1,\pm2,\pm3,....$ Trigger of coupled three singular points $\varphi_0=0$ and $\varphi_{1,3}=\pm\arccos\lambda$	$f(\lambda_s)=0$ $\lambda_s^4-2\lambda\lambda_s^3+\langle\lambda^2+\varepsilon-1\rangle\lambda_s^2+2\lambda\lambda_s-\lambda^2=0$ $\varphi_s=\arccos\lambda_s$	$\varphi_s=s\pi$, $s=0,\pm1,\pm2,\pm3,....$ 1* $\lambda>1$ Stable centre type points $\varphi_0=0$, $\varphi_{s=2k}=2k\pi$, $s=0,\pm1,\pm2,\pm3,....$ No stable saddle points $\varphi_1=\pi$, $\varphi_{s=2k+1}=(2k+1)\pi$, $s=0,\pm1,\pm2,\pm3,....$ 2* $\lambda<1$ No stable saddle points $\varphi_s=s\pi$, $s=0,\pm1,\pm2,\pm3,....$ Stable centre type points $\varphi_s=\pm\arccos\lambda+2s\pi$, $s=0,\pm1,\pm2,\pm3,....$ Trigger of coupled three singular points $\varphi_0=0$ and $\varphi_{1,3}=\pm\arccos\lambda$
Approximation of differential equation around singular point – a linearization as a phenomenological approximate mapping system dynamics in local area around singular poit	Trigger of coupled three singular points $\varphi_0=0$ and $\varphi_{1,3}=\pm\arccos\lambda$ m for $\lambda<1$ 1* $\varphi_0=0$, $\ddot{\varphi}+\Omega^2(\lambda-1)\varphi=0$ 2* $\varphi_{1,3}=\pm\arccos\lambda$ $\varphi\Rightarrow\varphi_{1,3}+\varphi$ $\ddot{\varphi}+\Omega^2(1-\lambda)\varphi=0$	$f(\lambda_s)=0$ $\lambda_s^4-2\lambda\lambda_s^3+\langle\lambda^2+\varepsilon-1\rangle\lambda_s^2+2\lambda\lambda_s-\lambda^2=0$ $\varphi_s=\arccos\lambda_s$ Trigger of coupled three singular points $\varphi_0=\varphi_{00}$ and $\varphi_{1,3}=\pm\arccos\lambda_{1,3}$ 1* $\varphi_0=\varphi_{00}$, $\ddot{\varphi}+\Omega^2\langle\lambda-\cos(\varphi_{00}+\varphi)\rangle\sin(\varphi_{00}+\varphi)-$ $-\Omega^2\varepsilon\cos(\varphi_{00}+\varphi)=0$ 2* $\varphi_{1,3}=\pm\arccos\lambda_{1,3}$ $\varphi\Rightarrow\varphi_{1,3}+\varphi$ $\ddot{\varphi}+\Omega^2\langle\lambda-\cos(\varphi_{1,3}+\varphi)\rangle\sin(\varphi_{1,3}+\varphi)-$ $-\Omega^2\varepsilon\cos(\varphi_{1,3}+\varphi)=0$	Trigger of coupled three singular points $\varphi_0=0$ and $\varphi_{1,3}=\pm\arccos\lambda$ 1* $\varphi_0=o$, $\ddot{\varphi}+\Omega^2(\lambda-1)\varphi\approx\Omega^2\lambda ctg\alpha\cos\Omega t$ 2* $\varphi_{1,3}=\pm\arccos\lambda$ $\varphi\Rightarrow\varphi_{s1,3}+\varphi$ $\ddot{\varphi}+\Omega^2(1-\lambda^2)\left[1+\frac{\lambda ctg\alpha}{\sqrt{1-\lambda^2}}\cos\Omega t\right]\varphi\approx\Omega^2\lambda ctg\alpha\cos\Omega t$ Mathieu-Hill type differential equation $\frac{d^2\varphi}{d\tau^2}+(\tilde{\lambda}+\tilde{\gamma}\cos\tau)\varphi=h\cos\tau$ $\omega=\Omega\sqrt{(1-\lambda^2)}$, $\tilde{\lambda}=1-\lambda^2$ i $\tilde{\gamma}=\lambda\sqrt{1-\lambda^2}ctg\alpha=h\sqrt{\tilde{\lambda}}$, $\tau=\Omega t$.
Approximation of differential equation solution around singular point – a linearization as a phenomenological approximate mapping system dynamics in local area around singular poit	1* No stable saddle type singular point $\varphi_0=0$, for $\lambda<1$ $\varphi(t)=ACh\Omega t\sqrt{\lambda-1}+BSh\Omega t\sqrt{\lambda-1}$ 2*Stable centre type singular point $\varphi_{1,3}=\pm\arccos\lambda$, for $\lambda<1$ $\varphi\Rightarrow\varphi_{1,3}+\varphi$ $\varphi(t)=A\cos\Omega t\sqrt{\lambda-1}+B\sin\Omega t\sqrt{\lambda-1}$	1* No stable saddle type singular point $\varphi_0=\varphi_{00}$, $\varphi(t)=AChqt+BShqt$ 2*Stable centre type singular point $\varphi_{1,3}=\pm\arccos\lambda_{1,3}$ $\varphi\Rightarrow\varphi_{1,3}+\varphi$ $\varphi(t)=A\cos pt+B\sin pt$	Trigger of coupled three singular points $\varphi_0=0$ and $\varphi_{1,3}=\pm\arccos\lambda$ 1* $\varphi_0=o$, $\varphi(t)\approx A\cos\Omega t\sqrt{\lambda-1}+B\sin\Omega t\sqrt{\lambda-1}+\frac{\lambda ctg\alpha}{(\lambda-2)}\cos\Omega t$ 2* $\varphi_{1,3}=\pm\arccos\lambda$ $\varphi(t)=Ae^{\mu t}p_1(t)+Be^{-\mu t}p_2(t)$ Mathieu-Hill type funvtions μ is characteristic exponent; $p_i(t), i=1,2$ are periodic functions with period 2π depending of parameters $\tilde{\lambda}$ and $\tilde{\gamma}$.

Table 4. Linear and non-linear phenomenological approximate mappings by use approximation of differential equations as well as approximations of solution in local area of system kinetic parameters

	Phenomenological approximate mappings by linearization - Linear phenomenological approximate mappings in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$	Phenomenological approximate mappings by nonlinear approximation - - Non-linear phenomenological approximate mappings in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$
Differential equations	$\frac{d\varphi}{dt}=f(\varphi,v)$; $\qquad \frac{dv}{dt}=f_1(\varphi,v)$	$\frac{d\varphi}{dt}=f(\varphi,v)$; $\qquad \frac{dv}{dt}=f_1(\varphi,v)$
Phenomenological approximate mappings of system differential equations around in local area around singular point $N_s(\varphi_s,v_s)$, $s=1,2,3,4....$ by $\varphi(=)\varphi_s+\varphi$, $v(=)v_s+v$	$\frac{d\varphi}{dt}=\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v$ $\frac{dv}{dt}=\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi+\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v$	$\frac{d\varphi}{dt}=\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v+\frac{1}{2!}\left(\frac{\partial^2 f(\varphi,v)}{\partial\varphi^2}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi^2+\frac{1}{2!}\left(\frac{\partial^2 f(\varphi,v)}{\partial v^2}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v^2+2\frac{1}{2!}\left(\frac{\partial^2 f(\varphi,v)}{\partial v\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi v+$ $+\frac{1}{3!}\left[\left(\frac{\partial^3 f(\varphi,v)}{\partial\varphi^3}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi^3+3\left(\frac{\partial^3 f(\varphi,v)}{\partial\varphi^2\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi^2 v+3\left(\frac{\partial^3 f(\varphi,v)}{\partial\varphi\partial v^2}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi v^2+\left(\frac{\partial^3 f(\varphi,v)}{\partial v^3}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v3\right]+....$ $\frac{dv}{dt}=\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi+\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v+\left(\frac{\partial^2 f_1(\varphi,v)}{\partial\varphi^2}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi^2+\left(\frac{\partial^2 f_1(\varphi,v)}{\partial v^2}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v^2+2\frac{1}{2!}\left(\frac{\partial^2 f_1(\varphi,v)}{\partial v\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi v+$ $+\frac{1}{3!}\left[\left(\frac{\partial^3 f_1(\varphi,v)}{\partial\varphi^3}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi^3+3\left(\frac{\partial^3 f_1(\varphi,v)}{\partial\varphi^2\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi^2 v+3\left(\frac{\partial^3 f_1(\varphi,v)}{\partial\varphi\partial v^2}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi v^2+\left(\frac{\partial^3 f_1(\varphi,v)}{\partial v^3}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}v3\right]+....$
Phenomenological approximate mappings of differential equations around in local area around singular point $N_s(\varphi_s,v_s)$, $s=1,2,3,4....$ by $\varphi(=)\varphi_s+\varphi$, $v(=)v_s+v$	$\frac{d^2\varphi}{dt^2}-\frac{d\varphi}{dt}\left\langle\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\right\rangle-$ $-\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi=0$	$\frac{d^2\varphi}{dt^2}-\frac{d\varphi}{dt}\left\langle\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\right\rangle-$ $-\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\varphi=F\left(\varphi.\frac{d\varphi}{dt}\right)$
Phenomenological approximate mappings of differential equation solution around in local area around singular point $N_s(\varphi_s,v_s)$, $s=1,2,3,4....$	1* for $\omega^2=-\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}>0$ $2\delta=-\left\langle\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\right\rangle>0$ approximate solution a* $\delta<\omega$ $\varphi(t)=e^{-\delta t}\left(A\cos t\sqrt{\omega^2-\delta^2}+B\sin t\sqrt{\omega^2-\delta^2}\right)$ Stable centre type singular point b* $\delta>\omega$ $\varphi(t)=e^{-\delta t}\left(ACht\sqrt{\delta^2-\omega^2}+BSht\sqrt{\delta^2-\omega^2}\right)$ No stable saddle type singular point 2* for $q^2=\left(\frac{\partial f(\varphi,v)}{v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}>0$ $\pm 2\delta=-\left\langle\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\right\rangle>0$	* for $\omega^2=-\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}>0$ $2\delta=-\left\langle\left(\frac{\partial f(\varphi,v)}{\partial\varphi}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}+\left(\frac{\partial f(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\left(\frac{\partial f_1(\varphi,v)}{\partial v}\right)\Big\vert_{\substack{\varphi=\varphi_s\\ v=v_s}}\right\rangle>0$ approximate solution a* $\delta<\omega$ $\varphi(t)=e^{-\delta t}\left(A\cos t\sqrt{\omega^2-\delta^2}+B\sin t\sqrt{\omega^2-\delta^2}\right)$ Stable centre type singular point Differential nonlinear approximate equation $\frac{d^2\varphi}{dt^2}+2\delta\frac{d\varphi}{dt}+\omega^2\varphi=F\left(\varphi.\frac{d\varphi}{dt}\right)$ By asymptotic method Krilov-Bogolybov-Mitropolyski Phenomenological approximate solution is proposed in the form: $\varphi e^{\delta t}=\tilde{x}=a\cos\psi+\varepsilon U_1(a,\psi,\tau)+\varepsilon^2U_2(a,\psi,\tau)+\dots$ $\frac{da}{dt}=\varepsilon A_1(a,\tau)+\varepsilon^2A_2(a,\tau)+\dots$ $\frac{d\psi}{dt}=p+\varepsilon B_1(a,\tau)+\varepsilon^2B_2(a,v)+\dots$ $\int_0^{2\pi}U_j(a,\psi,\tau)e^{i\psi}d\psi=0 \qquad j=1,2,\dots m$ $A_1(a.\tau)=-\frac{1}{2\pi p}\int_0^{2\pi}\tilde{\tilde{F}}\left(e^{-\tilde{\delta}\tau}\{a\cos\psi\},e^{-\tilde{\delta}\tau}\{-ap\sin\psi\}\right)e^{\tilde{\delta}\tau}\sin\psi d\psi$ $B_1(a.\tau)=-\frac{1}{2\pi pa}\int_0^{2\pi}\tilde{\tilde{F}}\left(e^{-\tilde{\delta}\tau}\{a\cos\psi\},e^{-\tilde{\delta}\tau}\{-ap\sin\psi\}\right)e^{\tilde{\delta}\tau}\cos\psi d\psi$ $\varepsilon\tilde{F}\left(\tilde{x}e^{-\delta t},\frac{d\tilde{x}}{dt}e^{-\delta t}\right)=\varepsilon\tilde{\tilde{F}}\left(\tilde{x},\frac{d\tilde{x}}{dt},\tau\right)$

Table 5. Example: Linear and non-linear phenomenological approximate mappings by use approximation of differential equation as well as approximations of solution in local area of system kinetic parameters

	Phenomenological approximate mappings by linearization - Linear phenomenological approximate mappings in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$	Phenomenological approximate mappings by nonlinear approximation - - Non-linear phenomenological approximate mappings in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$
Model		
Differential equations	$\ddot{\varphi}+\Omega^2\langle\lambda-\cos\varphi\rangle\sin\varphi-\Omega^2\varepsilon\cos\varphi=0$ or $\frac{d\varphi_2}{dt}=v$ $\frac{dv}{dt}=-\Omega^2\left\langle\frac{g}{R\Omega^2}-\cos\varphi_2\right\rangle\sin\varphi_2+\Omega^2\frac{r_{012}}{R}\cos\varphi_2$	$\ddot{\varphi}_2+\Omega^2\left(\frac{g}{\Omega^2 R}\cos\beta_0-\cos\varphi_2\right)\sin\varphi_2-\Omega^2\frac{r_{012}}{R}\cos\varphi_2=$ $=\frac{g}{R}\cos\varphi_2\sin\beta_0\sin\Omega t_1$ $\ddot{\varphi}+\Omega^2\langle\lambda-\cos\varphi\rangle\sin\varphi-\Omega^2\varepsilon\cos\varphi=0$
Phenomenological approximate mappings of system differential equations around in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$ by $\varphi(=)\varphi_s+\varphi$, $v(=)v_s+v$	$\frac{d\varphi}{dt}=v$ $\frac{dv}{dt}=-\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle\varphi$ or $\frac{d^2\varphi}{dt^2}+\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle\varphi=0$ Or $\ddot{\varphi}+\omega_{0,lin}^2\varphi=0$ $\frac{g}{R}\cos\varphi_2\sin\beta_0\sin\Omega t\approx\frac{g}{R}\sin\beta_0\sin\Omega t$	$\frac{d\varphi}{dt}=v$ $\frac{dv}{dt}=-\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle\varphi+\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\sqrt{1-\lambda_s^2}-4\lambda_s\sqrt{1-\lambda_s^2}\right\rangle-\frac{r_{012}}{R}\lambda_s\right\rangle\varphi^2$ $+\frac{\Omega^2}{3!}\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-8\lambda_s^2+4\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle\varphi^3+....$ or $\frac{d^2\varphi}{dt^2}+\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle\varphi-\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\sqrt{1-\lambda_s^2}-4\lambda_s\sqrt{1-\lambda_s^2}\right\rangle-\frac{r_{012}}{R}\lambda_s\right\rangle\varphi^2-$ $-\frac{\Omega^2}{3!}\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-8\lambda_s^2+4\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle\varphi^3+....=0$ Or $\ddot{\varphi}+\omega_{0,lin}^2\varphi=\kappa_2\varphi^2+\kappa_3\varphi^3$ $\frac{g}{R}\cos\varphi_2\sin\beta_0\sin\Omega t\approx\frac{g}{R}\left(1-\frac{1}{2!}\varphi^2+\frac{1}{4!}\varphi^4-\frac{1}{6!}\varphi^6......\right)\sin\beta_0\sin\Omega t$
Phenomenological approximate mappings of system kinetic parameter in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$ by $\varphi(=)\varphi_s+\varphi$, $v(=)v_s+v$	$\omega_{0,lin}=\Omega\sqrt{\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle}$	$\omega_{0,lin}^2=\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle$ $\kappa_2=\Omega^2\left\langle\left\langle\frac{g}{R\Omega^2}\sqrt{1-\lambda_s^2}-4\lambda_s\sqrt{1-\lambda_s^2}\right\rangle-\frac{r_{012}}{R}\lambda_s\right\rangle$ $\kappa_3=\frac{\Omega^2}{3!}\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-8\lambda_s^2+4\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle$
Phenomenological approximate mappings of differential equation solution around in local area around singular point $N_s(\varphi_s, v_s)$, $s=1,2,3,4....$	$\varphi(t)=a_0\cos\Phi(t)$ $\Phi(t)=\omega_{0,lin}t+\phi_0$	$\varphi(t)=a(t)\cos\Phi(t)$ $a(t)=a_0=const$, $\Phi(t)=\omega_{0,lin}t+\phi(t)$ $\phi(t)=\frac{\Omega\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-8\lambda_s^2+4\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle}{16\sqrt{\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle}}[a(t)]^2t+\phi(0)$ $\omega_{nel}=\Omega\sqrt{\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle}\left\{1+\frac{\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-8\lambda_s^2+4\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle}{16\left\langle\left\langle\frac{g}{R\Omega^2}\lambda_s-2\lambda_s^2+1\right\rangle+\frac{r_{012}}{R}\sqrt{1-\lambda_s^2}\right\rangle}a_0^2\right\}$
General form of differential equation and solution	$\ddot{x}(t)+2\delta\dot{x}(t)+\omega_0^2x(t)=0$ $x(t)=R_0e^{-\delta t}\cos(pt+\phi_0)$	$\ddot{x}(t)+2\delta\dot{x}(t)+\omega_0^2x(t)=\mp\tilde{\omega}_N^2x^3(t)$ $x(t)=R_0e^{-\delta t}\cos\left(pt-\frac{3}{16\delta p}\tilde{\omega}_{N1}^2R_0^2\left(e^{-2\delta t}-1\right)+\phi_0\right)$, for $p=\sqrt{\omega_0^2-\delta^2}$, $\delta\neq0$, $\varepsilon=0$, $\omega_0^2>\delta^2$

position of axis of moving mass particle along circle rotation are presented. In all, three phase portraits, each of corresponding models, a trigger of coupled three singular points is visible for taken system kinetic parameters. Phenomenological approximate mappings of nonlinear differential equations and corresponding solutions describing different models of Watt's regulator non-linear dynamics are, also, presented in Table 3. For details see References [2].

In Table 4, in general, linear and non-linear phenomenological approximate mappings by use linear and simple nonlinear approximation of differential equations as well as approximations of solution in local area of system kinetic parameters and for arbitrary nonlinear system dynamics with one degree of freedom are presented. For obtaining approximate solution of obtained non-linear approximation of nonlinear differential equation is possible to use Krilov-Bogolyubov-Mitropolski asymptotic method of nonlinear mechanics [31-33].

In Table 5, use an example of a heavy mass particle motion along rotate circle around vertical or skew positioned rotation axis, elements of mathematical non-linear phenomenology are presented. Linear and non-linear phenomenological approximate mappings by use approximation of differential equation as well as linear and nonlinear approximations of solution in local area of system kinetic parameters are presented.

From obtained elements of mathematical phenomenology as results of phenomenological approximate linear and nonlinear mappings, presented in Tables 7, 8 and 9, is visible an analogy with a intuitive possible conclusions in step, by step process, present mathematically through two or more steps in phenomenological approximate linear or nonlinear maps, starting by simple linear maps, and continue by simplest nonlinear phenomenological approximate map and continue with more and more complex nonlinear in step by step process. This can be process same as process of numerical iterations of numerical analysis and numerical experiment for obtaining solution of nonlinear differential equation or to obtain roots of non-linear equation. Then all numerical scientific computations can be select as phenomenological approximate maps.

All elements of mathematical phenomenology of presented models in Tables 3, 4 and 5 of a heavy mass particle along rotate circle around vertical or skew positioned, centric or eccentric axis of circle rotation are possible to use, in analogy, for analyze nonlinear dynamic properties and phenomena in dynamics of rigid body coupled rotations around two no intersecting axes, and also nonlinear dynamic of a two step redactor with deviation properties of mass distribution in two coupled gear disks.

VI. CONCLUDING REMARKS

On the basis of the elements of mathematical phenomenology, scientific results in World research progress, is possible to summarize and classify into standard dynamical models with analogous phenomena and analogous characteristic parameters as well as analogous methods of reseatchs..

Elements of *phenomenological precise or approximate mappings* on the basis of investigation series of similar types of nonlinear phenomena in global or local area of system dynamics using qualitative, structural or mathematical analogies and similarities is powerful research tools applicable in different area of sciences..

Then, one of main research task, nowadays, is a project of reduction of models of different disparate nature system on the basis of elements of mathematical phenomenology and corresponding qualitative, or structural, or mathematical analogies as well as by approximate phenomenological mappings around singular dynamic states. Integration knowledge on the basis of the elements of mathematical phenomenology will be basic knowledge kernel for future education of new university generations of students and researchers with larger scientific culture.

Task of investigation of different types of analogies is close with large knowledge about models of static and dynamics of the systems from different area of sciences. Also, for identification of analogous kinetic parameters in disparate nature system dynamics is coupled with *the capability to have a high level of intuition and intuitive recognition similar models and methods on the basis of identification of analogies.*

Author believe that this very important task, in present time, is actual for research and obtained results would lead to a systematic basis analogous models and methods, that can computerize as expert system must require each researcher. Expert base of qualitative, structural or mathematical or logic analogous models would be very useful to every researcher, as well as to the students to develop their research capabilities and intuitive thinking.

These days appear series of journals and articles with analogous research results without knowledge, that analogous results exists long time in other area of sconces. Than, it is necessary an analysis and transfer of knowledge from one to other different area of sciences.

Aim of two papers, present *"Elements of mathematical phenomenology: II. Phenomenological mapping"* and previous *"Elements of mathematical phenomenology: I. Mathematical and Qualitative analogies"*, based on Petrović's theory contain the interdisciplinary contents consisting of author's original research results in applications idea and theory presented by Mihailo Petrovic in the form of a review articles. The papers present some of elements of mathematical phenomenology and phenomenological mappings in investigation of disparate nature system dynamics use some singular examples in subjective choose bounder by author's previous published suitable research results.

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